AskDrCallahan Algebra II with Trigonometry

Solutions to Selected Problems

4th Edition
rev 081712

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Section 1.1

2. True

6. True

8. True

Section 1.2

2. The degree is 2.

29. 2x – 3{x + 2 [x- (x + 5)] + 1}

2x –3{ x + 2 [-5] + 1}

2x – 3{x – 10 + 1}

2x – 3{x –9}

2x –3x + 27

-x + 27

37. $(3u-2v)^2-(2u-3v)(2u+3v)$

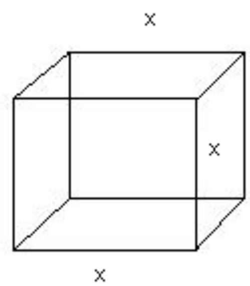
$$(3u-2v)^2=\left\lfloor 9u^2-6uv-6uv+4v^2\right\rfloor=\left\lfloor 9u^2-12uv+4v^2\right\rfloor$$

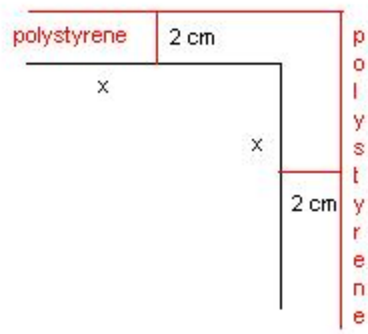
$$(2u-3v)(2u+3v)=\left\lfloor 4u^2+6uv-6uv-9v^2\right\rfloor=\left\lfloor 4u^2-9v^2\right\rfloor$$

$$\left\lfloor 9u^2-12uv+4v^2\right\rfloor-\left\lfloor 4u^2-9v^2\right\rfloor=9u^2-12uv+4v^2-4u^2+9v^2$$

$$=5u^2-12uv+13v^2$$

68. (View of the side of the cube when covered with polystyrene)





Volume of the Cube (VC) = x^3

Volume of the cube covered in polystyrene is (VC+P) = $(x+4)^3$ since you have the x value + 2 sides each 2cm.

Therefore:

$$V_C = x^3$$
$$V_{C+P} = (x+4)^3$$
$$V_P = V_{C+P} - V_C$$
$$V_P = (x+4)^3 - x^3 = \left[(x^2+8x+16)(x+4)\right] - \left[x^3\right]$$
$$= x^3 + 12x^2 + 48x + 64 - x^3$$
$$= 12x^2 + 48x + 64$$

<u>Section 1.3</u>

8. $a(3c+d) - 4b(3c+d) = (a-4b)(3c+d)$

25. $4x^2 - 20x + 25$
$(2x-5)(2x-5)$

Section 1.4

29. $$\frac{16-m^2}{m^2+3m-4} * \frac{m-1}{m-4} = \frac{(-m-4)(m-4)}{(m+4)(m-1)} * \frac{(m-1)}{(m-4)} =$$

$$\frac{(-m-4)(m-4)(m-1)}{(m+4)(m-1)(m-4)} = \frac{(-m-4)}{m+4} = -\frac{(m+4)}{(m+4)} = -1$$

39. $$\frac{1+\frac{2}{x}-\frac{15}{x^2}}{1+\frac{4}{x}-\frac{5}{x^2}} = \frac{1+\frac{2}{x}-\frac{15}{x^2}}{1+\frac{4}{x}-\frac{5}{x^2}} * \frac{x^2}{x^2} = \frac{x^2+2x-15}{x^2+4x-5} = \frac{(x+5)(x-3)}{(x+5)(x-1)} =$$

$$\frac{(x-3)}{(x-1)}$$

41. $$\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{\frac{1}{x+h}-\frac{1}{x}}{h} * \left(\frac{x(x+h)}{x(x+h)}\right) = \frac{x(x+h)\left(\frac{1}{x+h}\right)-\left(x(x+h)\left(\frac{1}{x}\right)\right)}{h(x(x+h))}$$

$$= \frac{x-(x+h)}{h(x(x+h))} = \frac{-h}{h(x(x+h))} = \frac{-1}{x(x+h)}$$

Section 1.5

33. $$\left(\frac{2x^{-3}y^2}{4xy^{-1}}\right)^{-2} = \frac{\left(2x^{-3}y^2\right)^{-2}}{\left(4xy^{-1}\right)^{-2}} = \frac{\left(4xy^{-1}\right)^2}{\left(2x^{-3}y^2\right)^2} = \frac{4^2x^2y^{-2}}{2^2x^{-6}y^4} = \frac{16x^2y^{-2}}{4x^{-6}y^4} = \frac{4x^8}{y^6}$$

46. $2^{(3^2)} = 2^9 = 512$ $\left(2^3\right)^2 = 8^2 = 64$

71. mass of earth = 6.1×10^{27} grams where 1 gram = 2.2×10^{-3} lbs

$$6.1\times10^{27}\,grams\left(\frac{2.2\times10^{-3}\,lbs}{1gram}\right) = 1.342 \times 10^{25} \text{ lbs}$$

Section 1.6

29. $\left(\frac{a^{2/3}b^{-1/2}}{a^{1/2}b^{1/2}}\right)^2 = \frac{a^{4/3}b^{-1}}{ab} = \frac{a^{4/3}}{ab^2} = \frac{a^{1/3}}{b^2}$

35. $(2x^{1/2} - 3y^{1/2})(2x^{1/2} + 3y^{1/2}) = 4x + 6x^{1/2}y^{1/2} - 6x^{1/2}y^{1/2} - 9y = 4x - 9y$

71. d = distance v = mph

d = .0212x$^{7/3}$

d = .0212(70)$^{7/3}$ = 428 ft

Section 1.7

29. $((xy)^{1/3})^{1/5} = (xy)^{1/15} = \sqrt[15]{xy}$

45. $\sqrt[6]{a^4(b-a)^2} = (a^4(b-a)^2)^{\frac{1}{6}} = a^{2/3}(b-a)^{1/3} = \sqrt[3]{a^2(b-a)}$

53. $\frac{\sqrt{2m}\sqrt{5}}{\sqrt{20m}} = \frac{\sqrt{2}\sqrt{m}\sqrt{5}}{\sqrt{20}\sqrt{m}} = \frac{\sqrt{10}}{\sqrt{20}} = \frac{\sqrt{5}\sqrt{2}}{\sqrt{5}\sqrt{4}} = \frac{\sqrt{2}}{2}$

95.
$$M = \frac{M_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{M_o}{\sqrt{\frac{c^2}{c^2}-\frac{v^2}{c^2}}} = \frac{M_o}{\sqrt{\frac{c^2-v^2}{c^2}}} = \frac{M_o\sqrt{c^2}}{\sqrt{c^2-v^2}} =$$
$$M = \frac{M_o c}{\sqrt{c^2-v^2}} \times \frac{\sqrt{c^2-v^2}}{\sqrt{c^2-v^2}} = \frac{M_o c\sqrt{c^2-v^2}}{c^2-v^2}$$

Section 2.1

13. $\frac{2}{y} + \frac{5}{2} = 4 - \frac{2}{3y}$

$$6y\left(\frac{2}{y}+\frac{5}{2}\right)=\left(4-\frac{2}{3y}\right)6y$$
$$12+15y=24y-4$$
$$16=9y$$
$$y=\frac{16}{9}$$

23. $\frac{3a-1}{a^2+4a+4}-\frac{3}{a^2+2a}=\frac{3}{a}\rightarrow\left(\frac{3a-1}{(a+2)(a+2)}-\frac{3}{a^2+2a}\right)*a(a+2)^2$

$=\frac{3}{a}*a(a+2)^2\,a(3a-1)-3(a+2)=3(a+2)^2\rightarrow -4a-6=12a+12$

$a=\frac{-9}{8}$

45. **trick is to multiply both sides by 12

$$\frac{2}{3}x-10=\frac{1}{4}x$$
$8x-120=3x$ 57.
$$5x=120$$
$$x=40$$

57. First, for every 100 meters in depth the temp increases 2.5 degrees. Since the units of x are to be in km, we note that every 1000 meters of change will result in a 25 degree change.

Next note that at 3 km deep the temp is already 30 degrees.

Therefore T = 30 + 25(x-3) degrees.

Section 2.2

15.

$$\frac{6}{5}a+\frac{3}{2}b=-4$$
$$\frac{6}{5}a=-4-\frac{3}{2}b$$
$$a=\frac{5}{6}\left(-4-\frac{3}{2}b\right)$$
$$a=\frac{-10}{3}-\frac{5}{4}b$$

$$\frac{1}{3}a-\frac{5}{4}b=1$$
$$\frac{1}{3}\left[\frac{-10}{3}-\frac{5}{4}b\right]-\frac{5}{4}b=1$$
$$-\frac{10}{9}-\frac{5}{12}b-\frac{5}{4}b=1$$
$$-\frac{5}{12}b-\frac{15}{12}b=1+\frac{10}{9}$$
$$-\frac{20}{12}b=\frac{9}{9}+\frac{10}{9}=\frac{19}{9}$$
$$-\frac{5}{3}b=\frac{19}{9}$$
$$b=-\frac{19}{15}$$

$$a=\frac{-10}{3}-\frac{5}{4}b=\frac{-10}{3}-\frac{5}{4}\left(\frac{-19}{15}\right)=\frac{-7}{4}$$

25. boat speed =20kph river speed=2kph

Therefore the boat moves 20-2=18kph upstream and 20+2=22kph downstream.

Let x be the distance traveled upstream and y be the distance traveled downstream (note that the distance is the same for both trips.)

Now time traveled upstream is x/18 hours and time downstream is y/22 hours.

Equations:

a) x=y (since the same distance is traveled)

b) $\frac{x}{18}+\frac{y}{22}=\frac{1}{4}$ (Use ¼ to have in terms of hours and not minutes where 15 minutes =1/4 hour.)

Therefore we have $\frac{x}{18}+\frac{x}{22}=\frac{1}{4}$ yields $x=2.475km$

33. Two equations are $40M + 20T = 4000$ or $T = \dfrac{4000 - 40M}{20} = 200 - 2M$

(Keyboards)

$$32M + 32T = 4000 \quad \text{(Screens)}$$

Where M is the Mexico plant and T is the Taiwan plant.

By substitution $32M + 32(200 - 2M) = 4000$ yields M=75 Hours.

Now $T = 200 - 2M = 200 - 2(75)$=50 Hours

Section 2.3

47. $$-4 \le \frac{9}{5}x + 32 \le 68$$

$$-36 \le \frac{9}{5}x \le 36$$
$$-20 \le x \le 20$$

68. $p + q < p - q$

$q < -q$ subtract p from each side

$2q < 0$
$q < 0$

77.
$$\begin{aligned} T &= 30 + 25(x-3) \\ 200 &= 30 + 25(x-3) \\ 200 &= 30 + 25x - 75 \\ 245 &= 25x \\ 9.8 &= x \end{aligned} \qquad \begin{aligned} 345 &= 25x \\ 13.8 &= x \end{aligned} \qquad 9.8 \le x \le 13.8$$

Section 2.4

91. $\left|\dfrac{x-45.4}{3.2}\right|\langle 1$

$$\frac{x-45.4}{3.2}\langle 1$$
$$x-45.4\langle 3.2$$
$$x\langle 3.2+45.4$$
$$x\langle 48.6$$

OR

$$\frac{x-45.4}{3.2}\rangle 1$$
$$x-45.4\rangle -3.2$$
$$x\rangle -3.2+45.4$$
$$x\rangle 42.2$$

$$42.2 < x < 48.6$$

Section 2.5

35. $$\frac{1}{2-\sqrt{-9}}=\frac{1}{2-3i}=\frac{1}{2-3i}\left(\frac{2+3i}{2+3i}\right)=\frac{2+3i}{4+6i-6i-9i^2}=\frac{2+3i}{4+9}=\frac{2+3i}{13}=\frac{2}{13}+\frac{3}{13}i$$

43. x^2-2x+2 and $x=1-i$

$$(1-i)^2-2(1-i)+2=1-i-i+i^2-2+2i+2=1-2i-1-2+2i+2=0$$

Section 2.6

33. $3w^2 + 4w + 3 = 0$

$$w^2 + \frac{4}{3}w + 1 = 0$$

$$w^2 + \frac{4}{3}w = -1$$

$$w^2 + \frac{4}{3}w + \left(\frac{2}{3}\right)^2 = -1 + \left(\frac{2}{3}\right)^2 = -\frac{5}{9}$$

$$\left(w + \frac{2}{3}\right)^2 = -\frac{5}{9}$$

$$w = -\frac{2}{3} - \sqrt{-\frac{5}{9}} = -\frac{2}{3} - i\frac{\sqrt{5}}{3}$$

41. $7n^2 = -4n$ (note that 0 will work as an answer!)

$$7n = -4$$

$$n = -\frac{4}{7}, 0$$

Remember quadratics should have 2 solutions!

87.

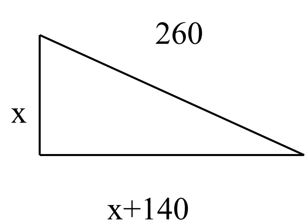


Figure describes distances of planes travel after 1 hour. Note right triangle.

Therefore

$$x^2 + (x+140)^2 = 260^2$$

$$x^2 + x^2 + 280x + 140^2 - 260^2 = 2x^2 + 280x - 48000 = 0$$

$$x^2 + 140x - 24000 = 0$$

Now use QE to get x=100,-240 miles. Since we are based on 1 hour, we see that x is also the speed in mph. But –240 does not make sense so we use speed of plane 1 = 100 mph and speed of plane 2 = 140 + 100 = 240 mph.

Section 2.7

33. $2t^{-4} - 5t^{-2} + 2 = 0$ (Note there will be 4 solutions.)

Let $x = t^{-2}$

$$2x^2 - 5x + 2 = 0$$
$$(x-2)(2x-1) = 0$$
$$x = 2, \frac{1}{2}$$
$$t = \pm\frac{1}{\sqrt{x}} = \pm\frac{1}{\sqrt{2}}, \pm\sqrt{2}$$

37. $4m + 8m^{1/2} - 5 = 0$

$$x = \sqrt{m}$$
$$x^2 = m$$
$$4x^2 + 8x - 5 = 0$$
$$(2x+5)(2x-1) = 0$$
$$x = \frac{-5}{2}, \frac{1}{2}$$
$$m = \frac{25}{4}, \frac{1}{4}$$

Now test the solutions to see if they really work. (critical since we had radicals)

$$test...m = \frac{25}{4}$$
$$4\left(\frac{25}{4}\right) + 8\sqrt{\frac{25}{4}} - 5 = 40 \neq 0$$ Does not work!

$$test\ldots m = \frac{1}{4}$$

$$4\left(\frac{1}{4}\right) + 8\sqrt{\frac{1}{4}} - 5 = 1 + 4 - 5 = 0$$

This one works!

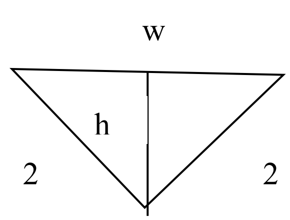
Therefore $m = \frac{1}{4}$.

47.

$$2x^{-2/5} - 5x^{-1/5} + 1 = 0$$
$$u = x^{-1/5}$$
$$2u^2 - 5u + 1 = 0$$
$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$$
$$= \frac{5 \pm \sqrt{25 - 8}}{4} = \frac{5 \pm \sqrt{17}}{4}$$

$$u^5 = \left(x^{-1/5}\right)^5$$
$$u^5 = x^{-1}$$
$$u^5 = \frac{1}{x}$$
$$x = \frac{1}{u^5}$$
$$x = \frac{1}{\left(\frac{5 \pm \sqrt{17}}{4}\right)^5} = \frac{4^5}{\left(5 \pm \sqrt{17}\right)^5}$$

51. The trick is to get the area of the triangle (end of the trough) and then multiply by 6 ft. Note that the end of the trough forms a rectangle (may not be a right triangle). Therefore the width of the trough is the other side of the triangle – called *w.* A perpendicular line from the bottom on the trough to the middle of the top forms two right triangles with sides w/2 and h and the hypotenuse=2.



By Pythagorean Theorem $\left(\frac{w}{2}\right)^2 + h^2 = 2^2 \rightarrow w = 2\sqrt{4-h^2}$.

The area of a triangle =1/2 Base*height or

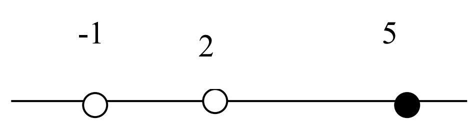
$$
\begin{aligned}
&A = \frac{1}{2}wh = \frac{1}{2} * 2 * \sqrt{4-h^2}\,h \\
&and \\
&V = 9 = 6 * A = 6 * \frac{wh}{2} = 6 * \sqrt{4-h^2}\,h \\
&therefore \\
&\frac{3}{2h} = \sqrt{4-h^2} \\
&\frac{9}{4h^2} = 4 - h^2 \\
&9 = 16h^2 - 4h^4 \\
&4h^4 - 16h^2 + 9 = 0 \\
&x = h^2 \rightarrow 4x^2 - 16x + 9 = 0 \\
&x = h^2 = \frac{16 \pm \sqrt{256 - 4*4*9}}{2*4} = \frac{16 \pm \sqrt{112}}{2*4} = 2 \pm 1.3228 \\
&w = 2\sqrt{4-h^2} = 3.65\,ft, 1.65\,ft
\end{aligned}
$$

Section 2.8

23. Step 1: Standard form results in $\frac{P}{Q} = \frac{x-5}{(x+1)(x-2)} \geq 0$

Step 2: x=5 is zero of P and x=-1,2 are zeros of Q

Step 3:



Test intervals	x=-2	P/Q = -7/4	-
	x=0	P/Q = +5/2	+
	x=4	P/Q = -1/10	-

x=6 P/Q = 1/28 +

Therefore equation is true when $-1 < x < 2$ or $x \geq 5$.

49. $|x^2 - 1| \leq 3$

$|x^2 - 1| - 3 \leq 0$

Solve two ways using the same similar steps as example 2.

a) $x^2 - 1 \leq 3$

$x^2 = 4$

$x = \pm 2$

b) $x^2 - 1 \geq -3$

$x^2 \geq -2$ This solution is complex and therefore not on the real number line.

So a) is the only possible values, now test the original equation

$|x^2 - 1| - 3 \leq 0$

to see when it is true/false around x=2 and x=-2.

x=-3 + false

x=0 - true

x=3 + false

Therefore $-2 \leq x \leq 2$

57. Start with putting in standard form and solving for 8000 units.

$$S = \frac{200t}{t^2 + 100}$$

$$8 \le \frac{200t}{t^2 + 100}$$

$$\frac{200t}{t^2 + 100} - 8 \ge 0$$

$$\frac{-8t^2 + 200t - 800}{t^2 + 100} \ge 0$$

Now solve for the zeros of the P and the Q. Notice that Q will not have real zeros (try it if you do not see it.) But the P (numerator) doe shave real zeros. Find via the quadratic equation.

Zeros for P are t=5,20

Now test to see where the equation $\frac{-8t^2 + 200t - 800}{t^2 + 100} \ge 0$ around the zeros.

t=4 yields negative number

t=6 yields positive number

t=21 yields negative number.

Therefore the sales will be equal to or over 8000 units in weeks from 5 to 20 or $5 \le t \le 20$.

Section 3.1

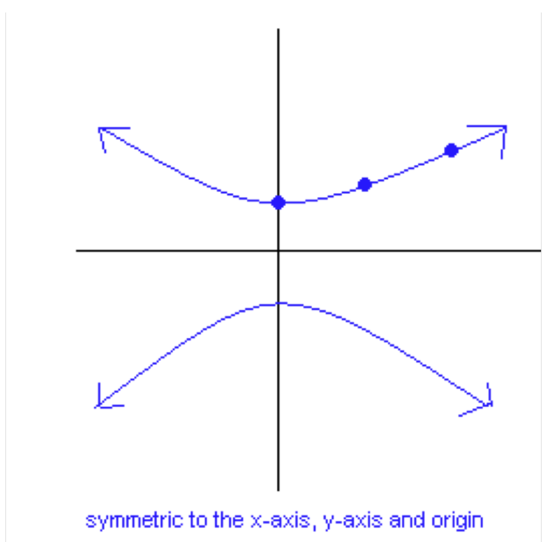
39.

$$4y^2 - x^2 = 1$$

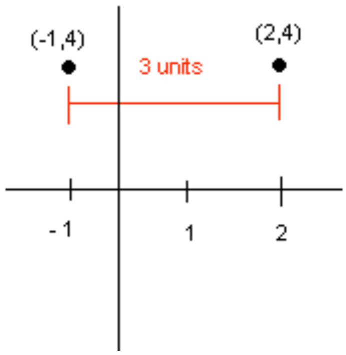
$$\frac{4y^2}{4} = \frac{1 + x^2}{4}$$

$$y^2 = \frac{1 + x^2}{4}$$

$$y = \pm\sqrt{\frac{1 + x^2}{4}}$$



53. This problem is easiest to see on a graph:

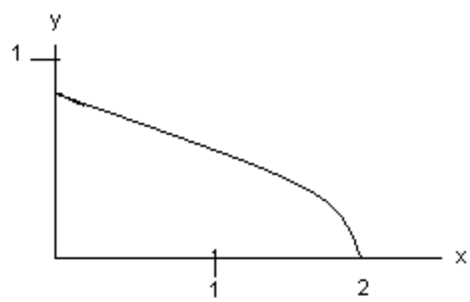


Answer: (2, 4)

91. $V = .5\left(\sqrt{2-x}\right)$

A)

V	x
.71	0
.5	1
0	2



B) Notice the graph is at a max (y-value) when x=0, which is when the pendulum is in mid-swing. At this time the velocity of the pendulum is at a maximum. When the pendulum gets to then end of a swing the velocity goes to 0.

Section 3.2

45. Since the value is (-2, -3) and it is perpendicular to the x-axis (straight up and down) this is a line where x=-2. In fact this is the equation --- x = -2.

71. First find slope $m = \frac{7-0}{0-(-2)} = \frac{7}{2}$

Then use the point slope formula

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{7}{2}(x - (-2))$$
$$y = \frac{7}{2}(x + 2)$$

Rearrange and multiply by 2 to get $7x - 2y = -14$.

86. Given: 5 psi on surface d(depth) = 0 feet p(pressure) = 15 psi

Solve for 30psi at 33 ft

A) We have two points, (0,15) and (33,30), therefore

$$slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30-15}{33-0} = \frac{15}{33}$$
$$y - y_1 = m(x - x_1)$$
$$y - 15 = \frac{15}{33}(x - 0)$$
$$y = \frac{15}{33}x + 15$$
$$d = x$$
$$p = y$$
$$therefore:$$
$$p = \frac{15}{33}d + 15$$

B) The question is asking: “how deep can he go?”

$p \leq 40\, psi$

$40 \leq \frac{15}{33} d + 15$

$40 - 15 \leq \frac{15}{33} d$

$d \leq 55\, ft$

<u>Section 3.3</u>

25. $2(F(-2) - G(-1))$

$$F(m) = 2m^2 + 3m - 1$$
$$G(n) = u^2 + u - 2$$
$$m = (-2)$$
$$n = (-1)$$
$$2[2(-2)^2 + 3(-2) - 1] - [(-1)^2 + (-1) - 2] =$$
$$2[2(4) - 6 - 1] - [1 - 1 - 2] =$$
$$2(8 - 6 - 1) - (-2)$$
$$= 4$$

67. Substitute $g(x) = 2 - x^2$ into equation to get

$$\frac{\lfloor 2-(3+h)^2 \rfloor - (2-3^2)}{h} = \frac{2-(9+6h+h^2)-2+9}{h} = \frac{-9-6h-h^2+9}{h} = -6-h$$

91. $V = x * Width * Length$

$Width = 8 - 2x$

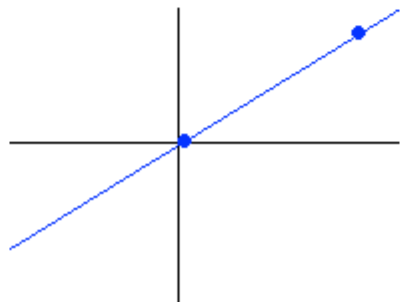
$Length = 12 - 2x$

$V(x) = x(8-2x)(12-2x)$

The domain is $0 \langle x \langle 4$ because you know that a volume measurement will be greater than 0, and therefore x is greater than 0. But if x is 4 or greater, then the volume will be equal to zero, or negative in value.

Section 3.4

77. A. $s = f(w) = mw + b$



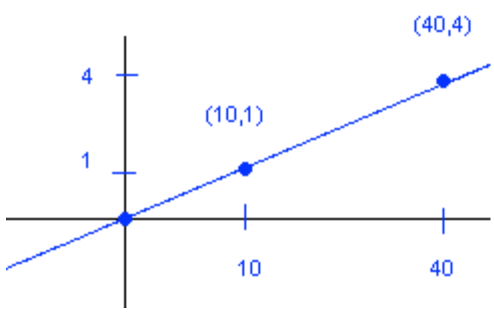
$(10,1) = (x_1, y_1)$
$(0,0) = (x_2, y_2)$
$m = \frac{0-1}{0-10} = \frac{-1}{-10} = \frac{1}{10}$
$b = 0$
$g = \frac{1}{10} w$

B) $f(15) = \frac{15}{10} = 1\frac{1}{2}''$

$F(30) = \frac{30}{10} = 3''$

C) $m = \frac{1}{10}$

D)



Section 3.5

5. Both domain and range are all real numbers. (Answer in back indicates real numbers with R.

39. $(f \circ g)(x) = f[g(x)] = \left|\frac{x}{x-3} + 2\right| = \left|\frac{x}{x-3} + \frac{2(x-3)}{x-3}\right| = \left|\frac{x+2x-6}{x-3}\right| = \left|\frac{3x-6}{x-3}\right| = 3\left|\frac{x-2}{x-3}\right|$

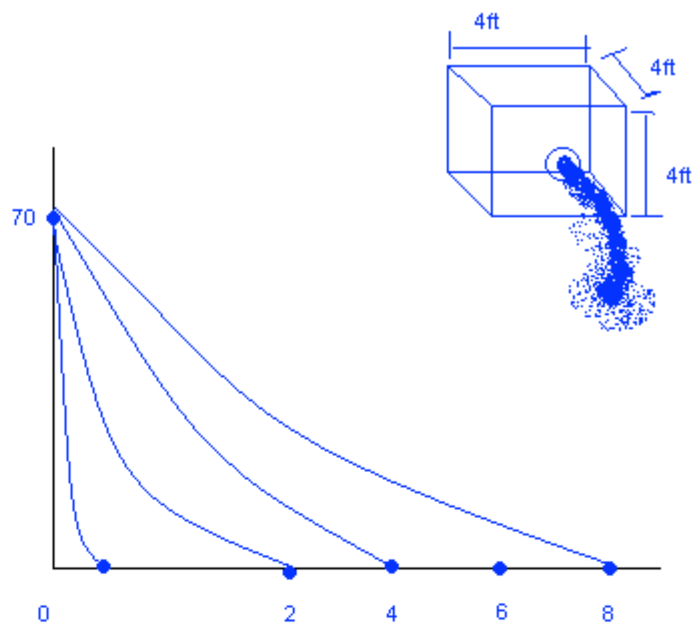
$D: x \neq 3$

$$(g \circ g)(x) = g[f(x)] = \frac{|x+2|}{|x+2|-3}$$

$D: x \neq 1, -5$

59. f(x)= x^4
g(x) = 2x – 7
h(x) = $f \circ g$

89.



V(t) = volume of water in the tank t minutes after water begins to flow: = $\frac{64}{c^2}(c-t)^2$

$0 \le t \le C$

<u>Section 3.6</u>

All are worked in the text.

Section 4.1

15.

$$
\begin{array}{r}
4y^2 - 4y + 1 \\
2y + 1 \overline{\big)\, 8y^3 - 4y^2 - 2y + 1} \\
-(8y^3 + 4y^2) \\
\hline
-8y^2 - 2y \\
-(-8y^2 - 4y) \\
\hline
2y + 1 \\
2y + 1 \\
\hline
0
\end{array}
$$

27. $P(x) = 2x^4 - 5x^3 + 2x^2 - 11x - 14$

Divide by x-3 using synthetic

$$
\begin{array}{r|rrrrr}
 & 2 & -5 & 2 & -11 & -14 \\
\hline
3 & 2 & 1 & 5 & 4 & -2
\end{array}
$$

Therefore R = -2

Check:

$$
\begin{aligned}
&P(3) = 2(3)^4 - 5(3)^2 + 2(3)^2 - 11(3) - 14 \\
&= 162 - 135 + 18 - 33 - 14 \\
&= -2
\end{aligned}
$$

57.

$$x^2 - 1 \overline{\big)\, x^4 + x^3 + x^2 - x - 2}$$

$$
\begin{array}{r}
x^2 \\
x^2 - 1 \overline{\big)\, x^4 + x^3 + x^2 - x - 2} \\
x^4 + 0 - x^2 \\
\hline
0 + x^3 + 2x^2 - x - 2
\end{array}
$$

$$
\begin{array}{r}
x^2 + x \\
x^2 - 1 \overline{\big)\, x^4 + x^3 + x^2 - x - 2} \\
x^4 + 0 - x^2 \\
\hline
0 + x^3 + 2x^2 - x - 2 \\
0 + x^3 + 0 - x \\
\hline
0 + 0 + 2x^2 + 0 - 2
\end{array}
$$

$$
\begin{array}{r}
x^2+x+2 \\
x^2-1\overline{\big)\, x^4+x^3+x^2-x-2} \\
x^4+0-x^2 \\
\hline
0+x^3+2x^2-x-2 \\
0+x^3+0-x \\
\hline
0+0+2x^2+0-2 \\
0+0+2x^2+0-2 \\
\hline
0
\end{array}
$$

Yields x^2+x+2 with R =0

Section 4.2

21. See theorem 6.

$P(x)=x^3-3x^2+2x-10$

$b:\pm2,\pm5,\pm10,\pm1$

$c:\pm1$

$\frac{b}{c}=\pm2,\pm5,\pm10,\pm1$

45. See Example 5. The goal is to factor the equation by finding the zeros, the places where it crosses the x-axis. Start by finding the possible rational zeros.

$$P(x)=x^4-2x^3-14x^2+30x+9 \Rightarrow \frac{b}{c}=\pm1,\pm3,\pm9$$

Using synthetic division test these until you find a zero. (where the remainder is zero.)

	1	-2	-14	30	9
1	1	-1	-14	16	25
3	1	1	-11	-3	0

therefore 3 is a zero, meaning x-3 is a factor and

$$P(x)=(x-3)(x^3-x^2-11x-3)=(x-3)Q(x)$$

now we factor the Q(x) the same way to get its factors. (Since it is 3rd order we cannot use the quadratic equation etc.)

first find the factors

$$P(x) = x^3 - x^2 - 11x - 3 \Rightarrow \frac{b}{c} = \pm 1, \pm 3$$

and

	1	-1	-11	-3
1	1	2	-9	-12
3	1	4	1	0

Therefore x-3 is a factor of Q(x) yielding

$$P(x) = (x-3)Q(x) = (x-3)(x-3)(x^2 + 4x + 1)$$

Now we have two zeros and can use the QE to get the last two.

$$x = \frac{-4 \pm \sqrt{4^2 - 4*1}}{2} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm \sqrt{3*4}}{2} = -2 \pm \sqrt{3}$$

$$P(x) = (x-3)Q(x) = (x-3)(x-3)(x + 2 - \sqrt{3})(x + +2 + \sqrt{3})$$

Or the zeros are at $x=3,3,$ $-2 \pm \sqrt{3}$

87. The original volume was 6 cubic ft. (3*2*1). Now add x to each side.

$$(x+1)(x+2)(x+3) = 6*10 = 60$$
$$P(x) = x^3 + 6x^2 + 11x - 54 = 0$$

Now find the solution to this equation using synthetic division. The possible zeros are

$$\frac{b}{c} = \pm 1, \pm 2, \pm 27, \pm 54$$

	1	6	11	-54
1	1	7	18	-36
-1	1	5	6	-60
2	1	8	27	0

Therefore $P(x) = (x-2)(x^2 + 8x + 27)$

If we try to find the roots of the other part using the QE we get

$$x = \frac{-8 \pm \sqrt{8^2 - 4*27}}{2} = \frac{-4 \pm \sqrt{16-108}}{2}$$

Without going any further we can tell these roots will be complex (negative square root) and therefore will make no sense. Therefore the only solution that makes physical sense is x=2ft.

Section 4.3

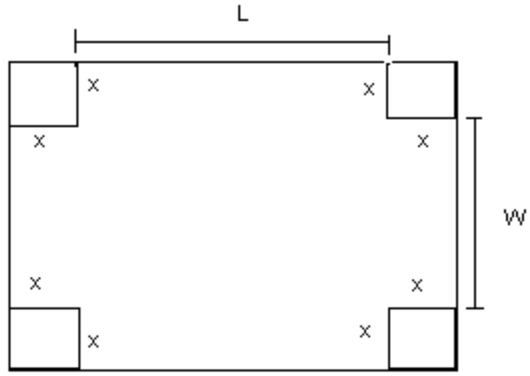
11. Review Thm 2 and example 2. The idea here is that all the squiggly stuff in the plot of the equation happens in a small range of x values, then the graph moves out to infinity or negative infinity at the two ends. The purpose of this exercise is the find the maximum and minimum values of x that surround the zeros. Notice in Figure 2 9a,b and c) that all the zeros of all three figures are in the range of x = –3 to 3. Theorem 2 is a trick to finding these values. (Not a trick that is used often, but it does work.)

So like Example 2 we will start at x=1 and go up until the upper bound is found. Then we will start at x = -1 and go down until the lower bound is found.

		1	-3	4	2	-9
	1	1	-2	2	4	-5
	2	1	-1	2	6	3
UB	3	1	0	4	16	39
	-1	1	-4	8	-6	-3
LB	-2	1	-5	14	-26	43

Upper x = 3, Lower x = -2

47.

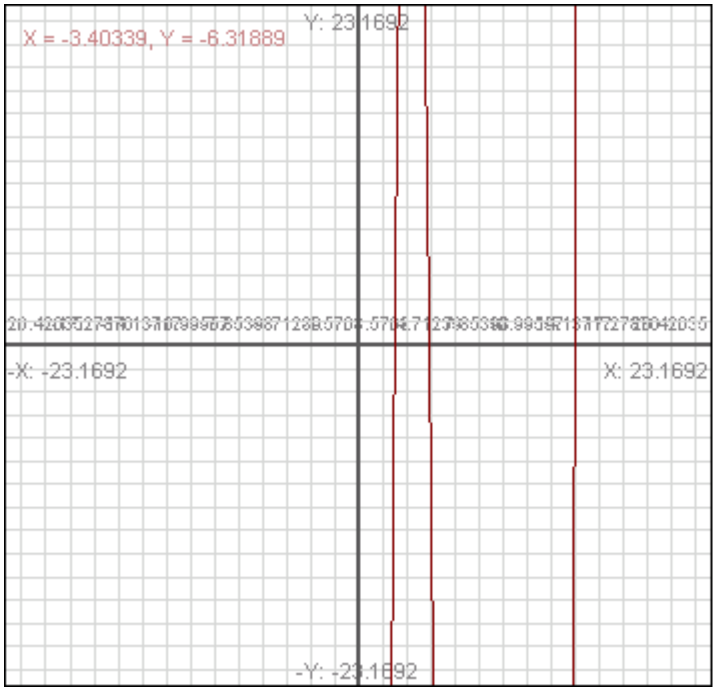


L = Length W = Width x = Height

Formula for Volume of a Cube = L(W)(x) = 600in^3

$$L = 24 - 2x$$
$$W = 18 - 2x$$
$$(24 - 2x)(18 - 2x)(x) = 600$$
$$432x - 48x^2 - 36x^2 + 4x^3 = 600$$
$$4x^3 - 84x^2 + 432x - 600 = 0$$

Now using a graphing calculator graph the function and estimate the solutions. See below the graph from Graphcalc (www.graphcalc.com). Three answers show up…. x=2.3, x=4.6, and x=14.1. Since 2 times 14 will be larger than the box this solution will not work, so the answer is 2.3 or 4.1 inches.



49. Notice from the figure that the two ends (if put together) would form a perfect sphere. The volume of a sphere is $\frac{4}{3}\pi r^3$ and therefore the volume of the sphere is $\frac{4}{3}\pi x^3$. (I had to look this up on Google – I do not remember everything either ;)

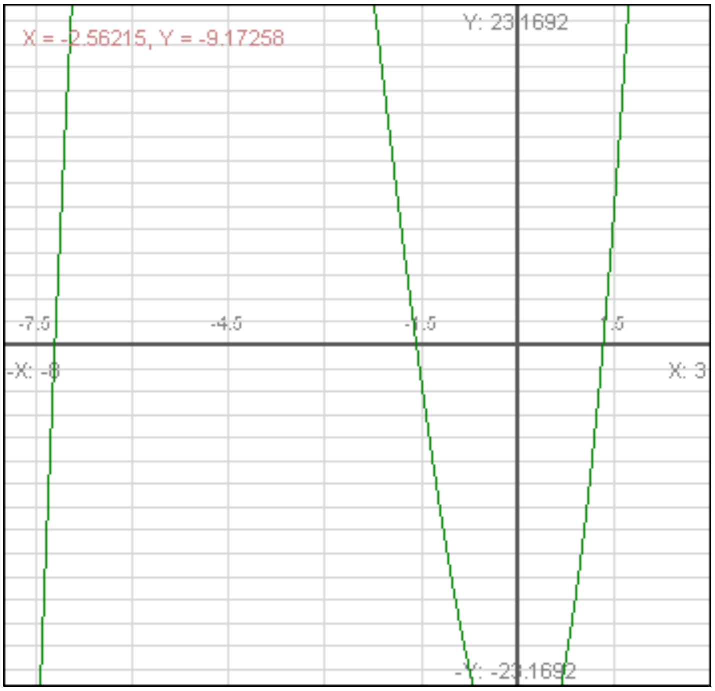
Now the volume of the rest of the tube is the area of a section, $\pi r^2 = \pi x^2$ times the length of the tube.

Therefore the total volume is

$$V = 10\pi x^2 + \frac{4}{3}\pi x^3 = 20\pi$$

$$V = 15x^2 + 2x^3 = 30$$

$$2x^3 + 15x^2 - 30 = 0$$



Therefore x=1.3 ft^3

**Note the answer in the back is different; the textbook author appears to have rearranged the equation wrong although he gets a similar answer.

<u>Section 4.4</u>

9. $$r(x) = \frac{x^2 - x - 6}{x^2 - x - 12} = \frac{(x-3)(x+2)}{(x-4)(x+3)}$$

$$d(x) = (x-4)(x+3) = 0$$

$$x = 4, -3$$

$$Domain: x \neq 4, -3$$

$$or$$

$$(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$$

$$n(x) = (x-3)(x+2) = 0$$

$$x = 3, -2$$

19.

$$t(x) = \frac{6x^4}{3x^2 - 2x - 5}$$

$$t(x) = \frac{n(x)}{d(x)} = \frac{6x^4}{3x^2 - 2x - 5}$$

$$d(x) = 0$$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0$$

$$3x - 5 = 0$$

$$x = 5/3$$

$$x + 1 = 0$$

$$x = -1$$

There are no horizontal asymptotes. (See theorem 2 and 3 to understand why.)
Vertical asymptotes at x = -1, 5/3.

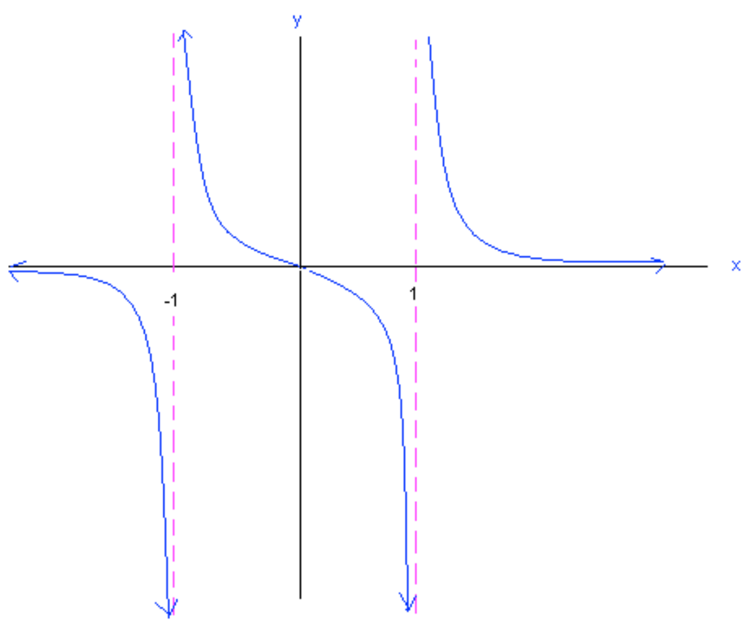
33.

$$f(x) = \frac{x}{x^2 - 1}$$

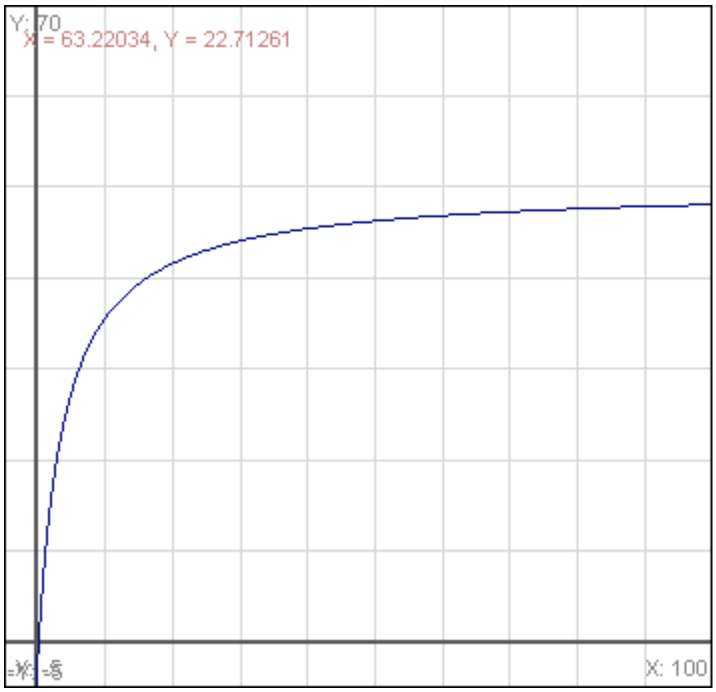
$$x\,\text{int} = 0$$

$$y\,\text{int} = 0$$

x	y
0	0
.1	-.01
.2	-.21
.3	-.33
.4	-.48
.5	-.67
.6	-.9
.7	-1.4
.8	-2.2
.9	-4.7
-.5	.6
-1	undefined
-.8	2.2
-.9	4.7
1.5	1.2
1.6	1.02
1.7	.899
1.8	.803
1.9	.728
2	.667
-1.5	-1.2
-1.6	-1.02
-1.7	-.899
-1.8	-.803
-1.9	-.728
-2	-.667



67. Note the equation has m=n (see Theorem 2). Also note that if x=0 then y=0 (y-intercept) and the x intercept (y=0) is x=0. So the graph goes through (0,0). It grows to a horizontal asymptote of y=50 (which is m/n). The only vertical asymptote would be at x=-4, which is not part of the problem. The graph below shows what happens. Note - an employee will produce up to 50 components per day and no more.



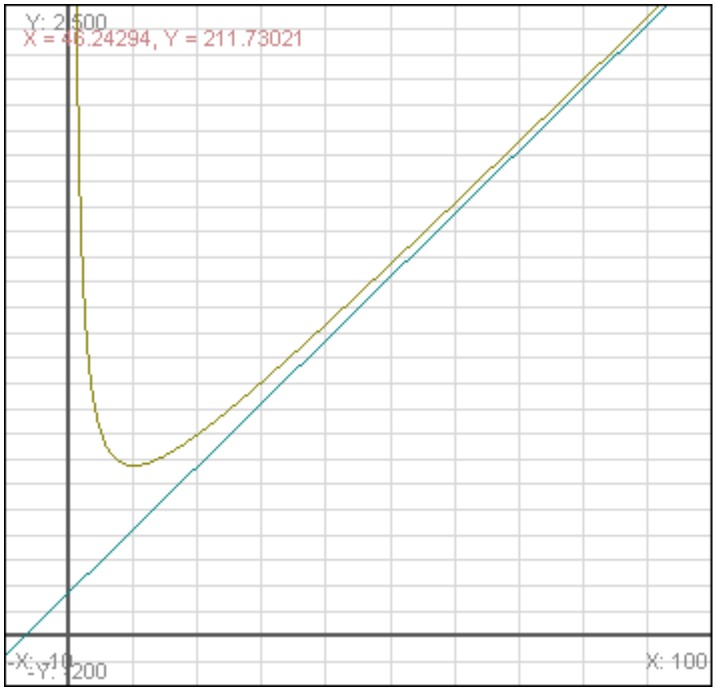
71. A) Divide to get

$$\overline{C(n)} = \frac{25n^2 + 175n + 2500}{n}$$

B) To find this graph the function first. Notice the y intercept =2500. Also notice that m=n+1 (see Theorem 3) and therefore an oblique asymptote may apply. To get the asymptote just divide to get

$$\overline{C(n)} = 25n + 175 + \frac{2500}{n}$$

Therefore, making $25n + 175$ the asymptote. Now you would just have to plot a few points to figure things out, so we will just look at the graph.



We can see the cost is at a minimum in about 10 years.

Section 5.1

27.

$$(2x+1)^3 = 8$$
$$(2x+1)^3 = 2^3$$
$$2x+1 = 2$$
$$2x = 1$$
$$x = \tfrac{1}{2}$$

61. Like example 3. Use d=2.4 days, and the initial population is 10.

$$P = P_0 2^{t/d} = 10 * 2^{t/2.4}$$

a) 1 week = 7 days
$P = 10 * 2^{7/2.4} = 75.5$ flies (or about 76)

b) $P = 10 * 2^{14/2.4} = 570$ flies

69. Best to try both and see. T=1/4 for one forth of a year.

a) $$A = P_0\left(1+\frac{r}{n}\right)^{nt} = 10000\left(1+\frac{0.089}{365}\right)^{365(1/4)} = \$10{,}225$$

b) $$A = 10000\left(1+\frac{0.09}{4}\right)^{4(1/4)} = \$10{,}225$$

Notice the amount is the same. Although the second has higher interest rate, the faster compounding gets the value up faster. For more information check out

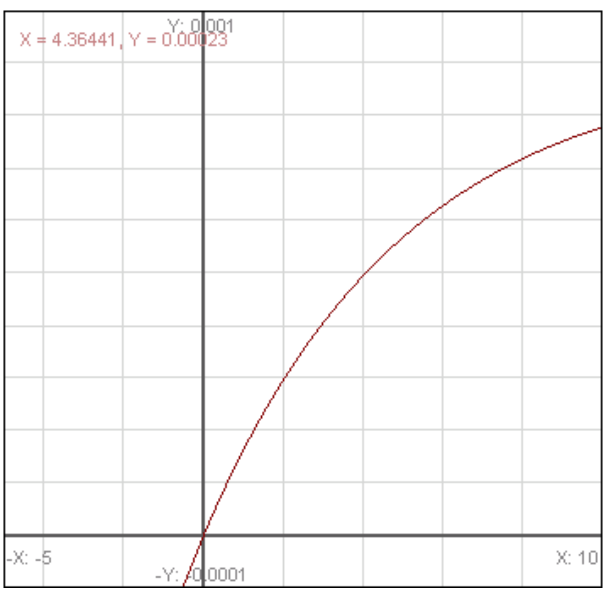
http://en.wikipedia.org/wiki/Compound_interest

"The most powerful force in the universe is compound interest."
Albert Einstein

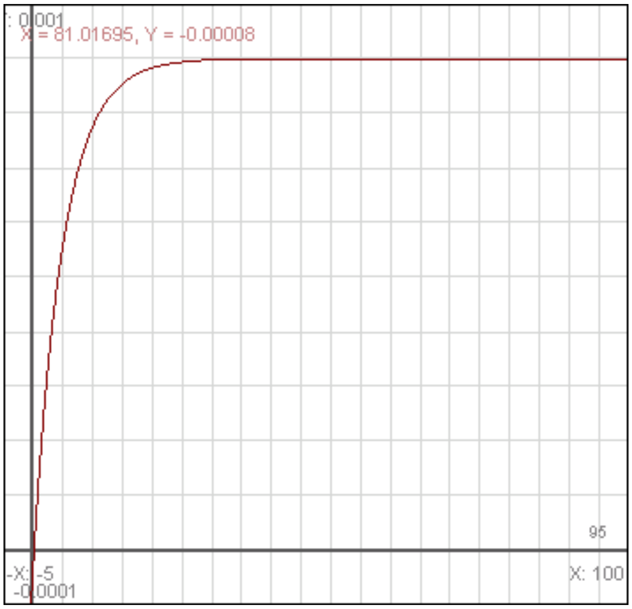
Section 5.2

27. $$\frac{e^{-x}(e^{x}-e^{-x})+e^{-x}(e^{x}+e^{-x})}{e^{-2x}} = \frac{1-e^{-2x}+1+e^{-2x}}{e^{-2x}} = \frac{2}{e^{-2x}} = 2e^{2x}$$

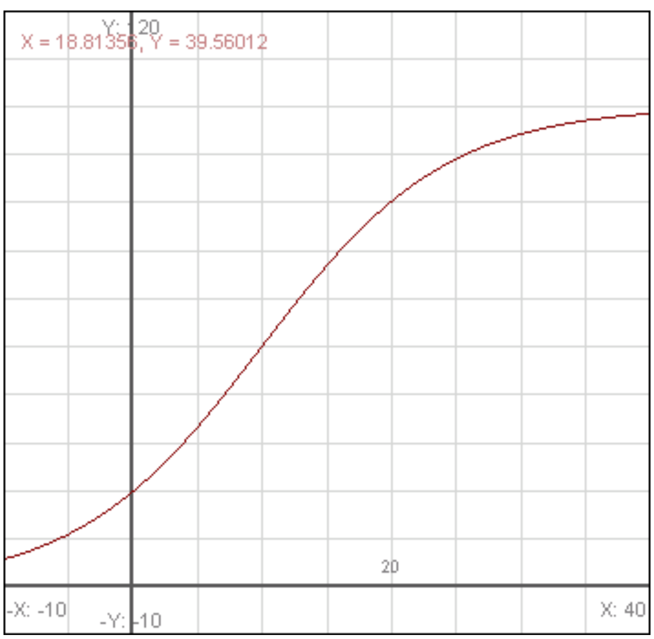
63. First graph the part for 0 to 10 seconds. (Note the graph is in terms of x and y but the x is seconds and the y is coulombs) You can see the exponential will go to zero, forcing the part inside the parenthesis to go to 1-0 or just 1.



Now to get the estimate we can either plug in a large value of seconds (try 100 seconds) or just graph out further.



65. Like before, just try graphing and see what you get. However, you can see as t increases, the exponential will go to 0, driving the denominator to 1.



In this one we see the max the equation will allow is 100 deer. (The equation is a model for the maximum amount of deer the island will support as well as the population growth.

Section 5.3

67. $\frac{1}{5}\left(2\log_b x + 3\log_b y\right) = \frac{1}{5}\left(\log_b x^2 + \log_b y^3\right) = \frac{1}{5}\left(\log_b (x^2 y^3)\right) = \log_b \sqrt[5]{x^2 y^3}$

105. I will put in terms of x and y. The book answer will swap the x and y at the final step. See section 3.6 if you do not remember how to do this.

$$g(x) = y = 3\log_e(5x-2)$$
$$\frac{y}{3} = \log_e(5x-2)$$
$$e^{\frac{y}{3}} = e^{\log_e(5x-2)}$$
$$e^{\frac{y}{3}} = 5x-2$$
$$e^{\frac{y}{3}} + 2 = 5x$$
$$\frac{1}{5}\left(e^{\frac{y}{3}} + 2\right) = x$$

Section 5.4

55. Like example 4 except the ratio of I/I_0 =1000. So

$$D = 10\log 1000 = 30dB$$

61. $v = c\ln\frac{Wt}{Wb} = 2.57\ln 19.8 = 7.67km/s$ (or about 17,126 miles per hour)

65. Problem looks complex, but quite simple. It is asking you to find the inverse of the equation given in problem 63.

$$pH = -\log[H^+] = 5.2$$
$$\log[H^+] = -5.2$$
$$10^{\log[H^+]} = 10^{-5.2}$$
$$H^+ = 10^{-5.2} = 6.3x10^{-6} moles/liter$$

Section 5.5

77. Solve by setting A=2*P_0

$$2P_0 = P_0\left(1+\frac{.15}{1}\right)^{1t}$$

$$2 = (1.15)^t$$

$$\ln 2 = \ln\left((1.15)^t\right) = t\ln 1.15$$

$$t = \frac{\ln 2}{\ln 1.15} = 4.96\,yrs$$

81. a) To get the dimmest, remember that L0 is the light flux. Therefore we will have L_0/L_0 which gives us

$$m = 6 - 2.5\log\frac{L}{L_0} = 6 - 2.5\log\frac{L_0}{L_0} = 6 - 2.5\log 1 = 6$$

b) The L/L_0 ratio provides how many times brighter one start is than another. So solve backwards for L/L_0.

$$m = 1 = 6 - 2.5\log\frac{L}{L_0}$$

$$-5 = -2.5\log\frac{L}{L_0}$$

$$\frac{-5}{-2.5} = \log\frac{L}{L_0} = 2$$

$$10^2 = 10^{\log\frac{L}{L_0}} = \frac{L}{L_0} = 100$$

Therefore a star with magnitude of 1 is 100 times brighter than a star with magnitude 6.

85. This problem is about carbon 14 dating. Some will have an issue with dating methods due to beliefs on age of the earth and creation. (I say some because even Christians and others who have pro-creation views will support old-earth views.)

However, the issue with the age-of-the-earth and creation has little to do with carbon 14 dating since carbon 14 can only give us dates back to thousands of years – even if it is accurate. See www.askdrcallahan.com/Products/AlgebraII.html for more information and links.

Now to solve the problem,

$$A = A_0 e^{-0.000124t}$$

$$0.1A_0 = A_0 e^{-0.000124t}$$

$$0.1 = e^{-0.000124t}$$

$$\ln(0.1) = -0.000124t$$

$$t = \frac{\ln 0.1}{-0.000124} = 18{,}569 yrs$$

87. Similar to Section 5.2 Problem 63.

$$q = 0.009(1 - e^{-0.2t})$$

$$0.0007 = 0.009(1 - e^{-0.2t})$$

$$0.7778 = 1 - e^{-0.2t}$$

$$0.2222 = e^{-0.2t}$$

$$-0.2t = \ln 0.2222 = -1.504$$

$$t = 7.52s$$

90. $I = I_0 e^{-kd}$

$$\frac{I_0}{2} = I_0 e^{-k(14.3)}$$

$$0.5 = e^{-k(14.3)}$$

$$\ln(0.5) = -k * 14.3$$

$$k = 0.0485$$

Section 6.1

67. A lot of information is given here, but the key elements are the angle and the arc length. So we get

$$\frac{s}{C} = \frac{\theta}{360}$$

$$\frac{500}{C} = \frac{7.5}{360}$$

$$C = \frac{500 * 360}{7.5} = 24{,}000 miles$$

Section 6.2

45. Label the other part of the right triangle x. (the side adjacent to beta.). Now the side of the large right triangle adjacent to alpha is d+x. Use the two right triangle to write equations.

$$\tan\alpha = \frac{h}{x+d}$$
$$\tan\beta = \frac{h}{x}$$
$$x = \frac{h}{\tan\beta}$$
$$\textit{therefore}$$
$$\tan\alpha = \frac{h}{\frac{h}{\tan\beta}+d}$$
$$\tan\alpha\left(\frac{h}{\tan\beta}+d\right) = h$$
$$\tan\alpha\frac{h}{\tan\beta} + d\tan\alpha = h$$
$$d\tan\alpha = h - \tan\alpha\frac{h}{\tan\beta} = h\left(1-\frac{\tan\alpha}{\tan\beta}\right)$$
$$h = \frac{d\tan\alpha}{1-\frac{\tan\alpha}{\tan\beta}} = \frac{d}{\frac{1}{\tan\alpha}-\frac{1}{\tan\beta}} = \frac{d}{\cot\alpha - \cot\beta}$$

55. Just use the second equation and plug in the given numbers.

$$g = \frac{v}{t\sin\theta} = \frac{4.1m/\sec}{3\sin 8^\circ \sec} = 9.8m/s^2$$

Note in particular the units are m/(s*s) or m/s^2 which are the units for acceleration.

Section 6.3

55. Notice the first part is simple and by definition.

a) $\theta = \frac{s}{r} = \frac{7}{4} = 1.75 rad$

b) Now to find the point P(a,b) we need to find the x and y coordinates. Notice below the pink line we have a triangle with angle

$\pi - 1.75 = 1.39 rad$ which is the reference triangle. (Discussed in the next section)

So we have a triangle with angle1.39 radians and hypotenuse of 4.

$$x = a = 4 * \cos(1.39) = 0.713$$
$$y = b = 4 * \sin(1.39) = 3.936$$

Now see that the x value must be negative we get P(-0.713,3.936).

59. a) Just plug in the number for the angle to get

theta = 0, I =k
theta = 30, I =0.866k
theta =60, I =0.5k

b) I is max when theta =0, in which case I=k. Therefore we want to find when I=k/4.

$$0.25k = k\cos\theta$$
$$0.25 = \cos\theta$$
$$\theta = \cos^{-1}(0.25) = 75.5°$$

Section 6.4

61. $\sin\theta = \frac{-1}{\sqrt{2}}$

The smallest angle that is the solution is in quadrant III (remember the sin goes with the y value – so y must be negative.) So $\frac{-1}{\sqrt{2}}$ is the reference angle where the y side is 1 and the hypotenuse is $\sqrt{2}$.This is a 45 degree triangle. Therefore 180 + 45 = 225 degrees.

Section 6.5

31. If you travel 55.33 units clockwise, we have made 8+ rotations.

$$\frac{55.33}{2\pi} = 8.806$$

So all we have to worry about is the last part of the rotation after the 8th. So subtract away the other to get

$$55.33 - 8 * 2\pi = 5.06452 rads$$

just remember we are moving in the counterclockwise (positive) direction and have gone from 0 to 5.06452 radians. Since we are on the unit circle

$$x = \cos(5.06452) = 0.345$$
$$y = \sin(5.06452) = -0.939$$

33. Count out $\frac{\pi}{6}$ (30 degrees) five times. You get 150 degrees, or in quad II. The reference angle is 180-150=30degrees ($\frac{\pi}{6}$) and since it is the sine function the y value will be positive. Therefore

$$\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$$

51. $$\frac{1-\sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\tan^2 x} = \cot^2 x$$

Section 6.6

9. Easiest thing to do is put all in terms of sine and cosine.

a) $y = \cos x$ - works for all values so no vertical asymptotes.

b) $y = \tan x = \dfrac{\sin x}{\cos x}$ has vertical asymptotes when $\cos x = 0$ or when

$x = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}$.

c) $y = \csc x = \dfrac{1}{\sin x}$ has vertical asymptotes when $\sin x = 0$ or when

$x = 0, \pm\pi, \pm 2\pi$

21. If f(x) is periodic then g(x)=5f(x) is also periodic since you are only multiplying the value by 5, and therefore will only increase the y value 5 times for every x value.

Section 6.7

51. $y = 3.5\sin\left[\dfrac{\pi}{2}(t + 0.5)\right] = 3.5\sin\left[\dfrac{\pi}{2}t + \dfrac{\pi}{4}\right]$

Amplitude = 3.5

$$\frac{\pi}{2}t + \frac{\pi}{4} = 0 \qquad\qquad \frac{\pi}{2}t + \frac{\pi}{4} = 2\pi$$
$$t = -0.5 \qquad\qquad t = -0.5 + 4$$

Phase Shift = -0.5
Period = 4

Graph in text.

73. First note the in problem 71 he uses feet in the equation – we will use inches. Therefore the amplitude is 8 inches as given. So the equation will be of the form

$y = 8\cos Bt$

now since P=0.5 secs

$B = \dfrac{2\pi}{0.5} = 4\pi$ (here we are using the equation $Bx + C = 2\pi$ where C=0 since we are asked to find in a form without C.)

Now the equation looks like

$y = 8\cos 4\pi t$

and we need to test the point given t = 0 and y = -8inches.

8cos(0)=8. Therefore to make it negative (notice it says 8 inches below) we need to make the final equation

$y = -8\cos 4\pi t$ inches.

74. Similar to the problem above with C=0 s

$$P = \frac{2\pi}{B} = \frac{1}{60}$$
$$B = 2\pi * 60 = 120\pi$$

$$y = 110\cos(120\pi t)$$

Test given the point t=0 yields y=110. Therefore works.

79. Similar to problems 73 and 74. The shadow is at a minimum (or zero) when theta =0 and at a max at 90 degrees - which sounds like sine function. The radius of the disk is 3, which in plies the showdown will reach +3 to –3 as it rotates.

The key issue is the 3 rev/ sec. What we care about is how many seconds it takes to repeat or go around 1 time, which is the inverse.

$$\frac{1\sec}{3rev}$$

therefore like before (since we have no C value)

$$P = \frac{2\pi}{B} = \frac{1}{3}$$
$$B = 2\pi * 3 = 6\pi$$

So we have $y = 3\sin(6\pi t)$.

Section 6.8

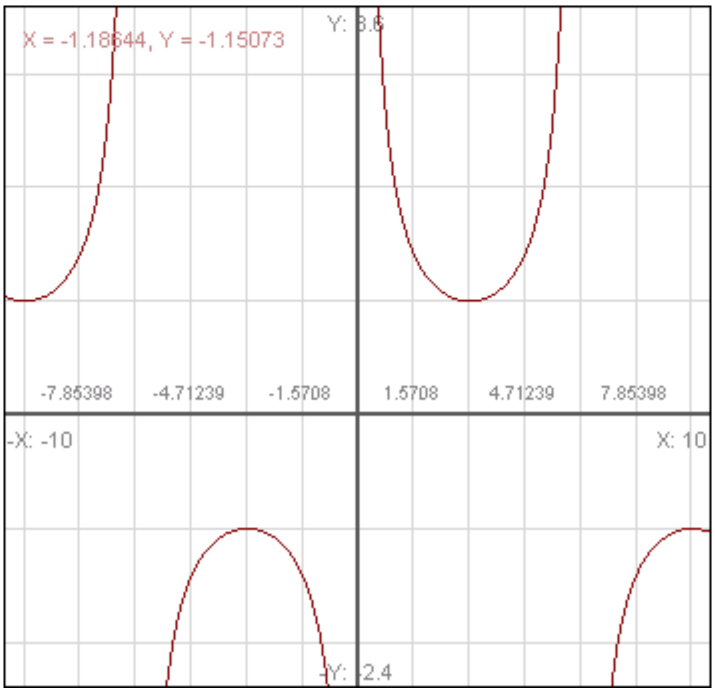
5. Like example 1

$$Bx + C = 0 \qquad\qquad Bx + C = 2\pi$$

$$\frac{1}{2}x = 0 \qquad\qquad \frac{1}{2}x = 2\pi$$

$$x = 0 \qquad\qquad x = 4\pi$$

Phase Shift = 0 Period = 4π



11. Notice the tan and cotangent repeat after pi and not 2*pi.

$$y = tax(2x + \pi)$$

$$Bx + C = 0 \qquad\qquad Bx + C = \pi$$

$$2x + \pi = 0 \qquad\qquad 2x + \pi = \pi$$

$$x = -\frac{\pi}{2} \qquad\qquad x = \frac{-\pi}{2} + \frac{\pi}{2}$$

Phase Shift = $-\frac{\pi}{2}$ Period = $\frac{\pi}{2}$

Graph in text.

31. a) Start by writing an equation for the triangle in terms of c.

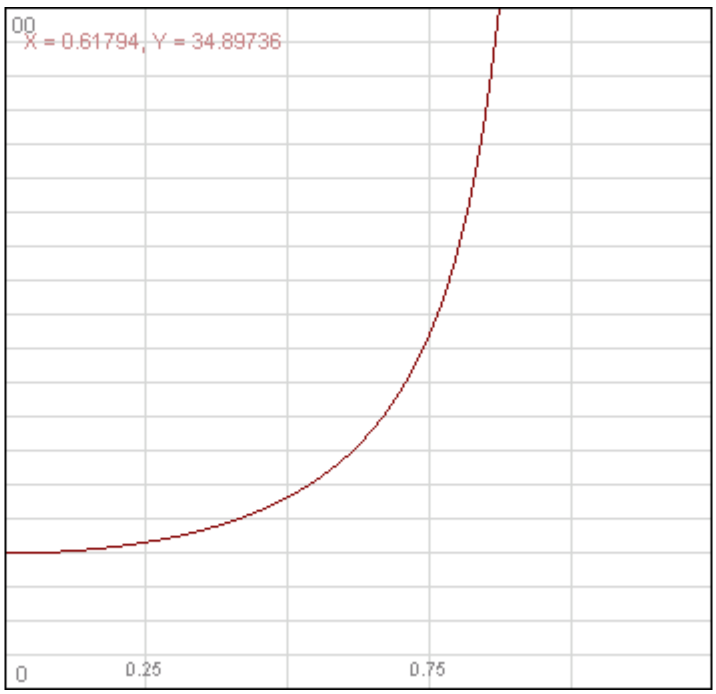
$$\cos\theta = \frac{20}{c}$$

$$c = \frac{20}{\cos\theta} = 20\sec\theta$$

and since theta was given we have

$$c = 20\sec\frac{\pi t}{2}$$

b)



c) As t approaches 1 the lamp approaches 90 degrees, the length c approaches infinity (theoretically).

<u>Section 6.9</u>

All textbook solutions are clear.

<u>Section 7.1</u>

11. $$\frac{\cos x - \sin x}{\sin x \cos x} = \csc x - \sec x$$

Start with the more complex side (the left side) and try to simplify to the right side. Therefore

$$\frac{\cos x-\sin x}{\sin x\cos x}=\frac{\cos x}{\sin x\cos x}-\frac{\sin x}{\sin x\cos x}=\frac{1}{\sin x}-\frac{1}{\cos x}=\csc x-\sec x$$

37. Show that $\dfrac{1-(\sin x-\cos x)^2}{\sin x}=2\cos x$

$$\frac{1-(\sin x-\cos x)^2}{\sin x}=\frac{1-(\sin^2 x-2\sin x\cos x+\cos^2 x)}{\sin x}=\frac{1-(\sin^2 x+\cos^2 x-2\sin x\cos x)}{\sin x}$$
$$=\frac{1-(1-2\sin x\cos x)}{\sin x}=\frac{2\sin x\cos x}{\sin x}=2\cos x$$

43. Show that $\tan^2 x-\sin^2 x=\tan^2 x\sin^2 x$

Start with the left side

$$\tan^2 x-\sin^2 x=\frac{\sin^2 x}{\cos^2 x}-\sin^2 x=\sin^2 x\left(\frac{1}{\cos^2 x}-1\right)=\sin^2 x\left(\frac{1}{\cos^2 x}-\frac{\cos^2 x}{\cos^2 x}\right)$$
$$=\sin^2 x\left(\frac{1-\cos^2 x}{\cos^2 x}\right)$$

now note that $\sin^2 x+\cos^2 x=1$ can be arranged as $\sin^2 x=1-\cos^2 x$ and therefore

$$\sin^2 x\left(\frac{1-\cos^2 x}{\cos^2 x}\right)=\sin^2 x\left(\frac{\sin^2 x}{\cos^2 x}\right)=\sin^2 x\tan^2 x=\tan^2 x\sin^2 x$$

47. Show that $\ln(\tan x)=\ln(\sin x)-\ln(\cos x)$
See pg 380, Theorem 1, number 6.

$$\ln(\sin x)-\ln(\cos x)=\ln\left(\frac{\sin x}{\cos x}\right)=\ln(\tan x)$$

<u>Section 7.2</u>

25. Hint use x=45 and y =30 with sin(x-y)

29. Hint: You do not need to find the angles, just the ratios. Like example 4.

$\sin x = \frac{-3}{5}$ $\qquad$ $\sin y = \frac{\sqrt{8}}{3}$

$\cos x = \frac{4}{5}$ $\qquad$ $\cos y = \frac{1}{3}$

$\tan x = \frac{-3}{4}$ $\qquad$ $\tan x = \sqrt{8}$

33. Show that $\cos 2x = \cos^2 x - \sin^2 x$.

$$\cos^2 x - \sin^2 x = \cos x \cos x - \sin x \sin x = \cos(x + x) = \cos(2x)$$

Using the sum identity for cos(x+y) where x=y.

Section 7.3

15. $(\sin x + \cos x)^2 = 1 + \sin 2x$

$$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x = 1 + 2\sin x \cos x = 1 + \sin 2x$$

17. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$\frac{1}{2}(1 - \cos 2x) = \frac{1}{2}\left[1 - \left(1 - 2\sin^2 x\right)\right] = \frac{1}{2}\left[1 - 1 + 2\sin^2 x\right] = \sin^2 x$$

21. $\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$ (By definition – see page 547)

69. Notice we have two right triangles. Write the relationships.

$$\cos 2\theta = \frac{7}{x} = 2\cos^2 \theta - 1$$

where

$\cos\theta = \frac{7}{8}$ (from the smaller triangle)s

Therefore substitute to get

$$\cos 2\theta = \frac{7}{x} = 2\left(\frac{7}{8}\right)^2 - 1 = 2\frac{49}{64} - 1 = \frac{49}{32} - \frac{32}{32} = \frac{17}{32}$$

Therefore

$$x = 7 * \frac{32}{17} = 13.176m$$

Now the angle is

$$\theta = \cos^{-1}\frac{7}{8} = 28.955^{\circ}$$

79. For part a) $d = \dfrac{v_0^2 \sin 2\theta}{32}$

b) to find the maximum d, the numerator must be the largest it can be. Since sin(x) will be at most 1, we need to solve for that.

$$\sin 2\theta = 1$$

True when the angle is 45 degrees.

Section 7.4

17. $\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$ --- by definition --- see page 552.

23. $$\frac{\sin x - \sin y}{\cos x - \cos y} = -\cot\frac{x+y}{2}$$

$$\frac{\sin x - \sin y}{\cos x - \cos y} = \frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}} = -\frac{\cos\frac{x+y}{2}}{\sin\frac{x+y}{2}} = -\cot\frac{x+y}{2}$$

Section 7.5

21. $$2\sin^2\theta + \sin 2\theta = 0, all\theta$$
$$2\sin^2\theta + 2\sin\theta\cos\theta = 0$$
$$2\sin\theta(\sin\theta + \cos\theta) = 0$$

Solution where $2\sin\theta = 0$ or $\sin\theta + \cos\theta = 0$. Notice $\sin\theta + \cos\theta = 0$ when one is negative of the other – at the 45 degree angles. True at 135 and 310 degrees.

Therefore

$\theta = 180k, 135 + 360k, 310 + 360k,$ where k is an integer

Or can be rewritten as

$\theta = 180k, 135 + 180k,$ where k is an integer

25. $2\cos^2\theta + 3\sin\theta = 0, 0° \le \theta < 360.$

Rearrange as a quadratic.

$$2(1 - \sin^2\theta) + 3\sin\theta = 0$$
$$2 - 2\sin^2\theta + 3\sin\theta = 0$$
$$2\sin^2\theta - 3\sin\theta - 2 = 0$$
$$x = \sin\theta$$
$$2x^2 - 3x - 2 = 0$$
$$x = \frac{8}{4}, -\frac{2}{4} = 2, -\frac{1}{2}$$

Since $x=2$ will not work we use $x = -\frac{1}{2}$ and solve for $x = -\frac{1}{2} = \sin\theta$

$\theta = 210°, 330°$

31. $6\sin^2\theta + 5\sin\theta = 6, 0° \le \theta \le 90.$

Setup as a quadratic -- $6\sin^2\theta + 5\sin\theta - 6 = 0$

And let $x = \sin\theta$ to make a simpler equation of

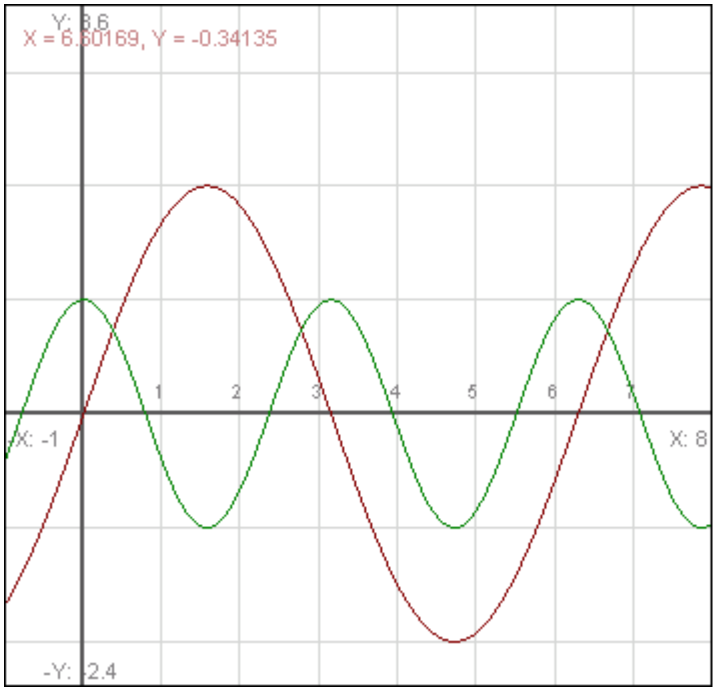
$6x^2 + 5x - 6 = 0$

Solve to get $x = \frac{8}{12}, -\frac{18}{12}$ (I used the quadratic equation to get this)

Now solve $x = \sin\theta$ where $\frac{8}{12} = \sin\theta$ (Note the other solution will not work since it is not within $-1 \le x \le 1$.

Therefore, $\theta = 41.81°$

43. Plotting we get



And using our graphing tools we see the intersections at

x=0.375, y=0.732

x=2.767, y= 0.732

63. Rearrange a bit and solve for

$$-10 = 30\sin 120\pi t$$
$$-\frac{1}{3} = \sin 120\pi t = \sin x$$
$$x = -0.3398 rad, \pi + 0.3398 rad$$

The smallest positive will be the answer in Quad III. So

$$x = \pi + 0.3398 = 120\pi t$$
$$t = \frac{\pi + 0.3398}{120\pi} = 0.009235 \sec$$

65. Hint : Solve $0.4I = I\cos^2\theta$

71. Part a) the goal is to find L, which is $L = 2\theta R$

write equations as you see them. You should get

$\tan\theta = \dfrac{a}{R-b}$ and $R^2 = (R-b)^2 + a^2$

Solving the second we can get R.

$$R = \frac{b^2 + a^2}{2b} = 7.3mm$$

Now we can find the angle by

$$\tan\theta = \frac{a}{R-b} = 1.14583$$
$$\theta = 0.853255 radians$$

And now we find L by

$$L = 2\theta R = 2*0.853255*7.3 = 12.7545mm$$

Part b) Notice we have a few equations from part a.

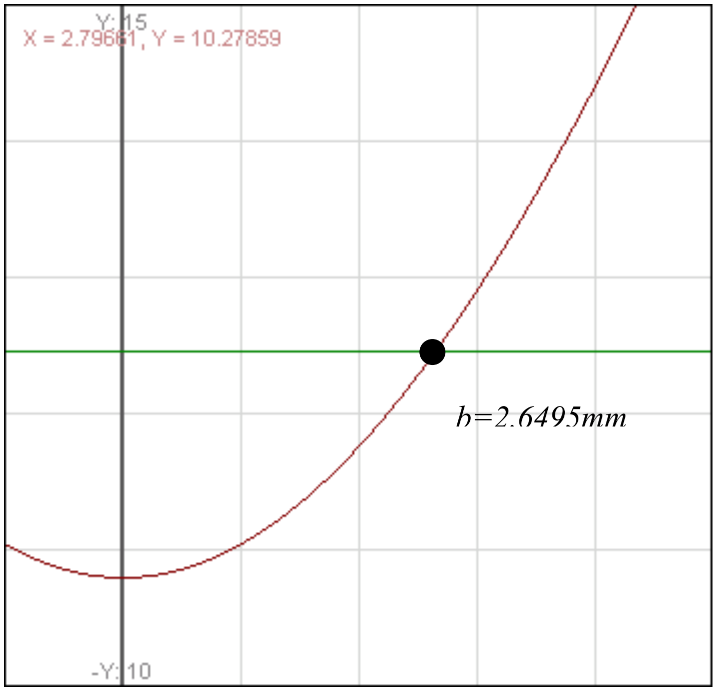
$$L = 2\theta R = 2*\sin^{-1}\left(\frac{a}{R}\right)*R = 2*\sin^{-1}\left(\frac{a}{\frac{b^2+a^2}{2b}}\right)*\frac{b^2+a^2}{2b}$$

$$L = 2*\sin^{-1}\left(\frac{2ab}{b^2+a^2}\right)*\frac{b^2+a^2}{2b}$$

For this part we have a few values and need to solve for *b*.

$$12.4575 = 2*\sin^{-1}\left(\frac{2(5.4)b}{b^2+(5.4)^2}\right)*\frac{b^2+(5.4)^2}{2b}$$
$$12.4575 = 2*\sin^{-1}\left(\frac{10.8b}{b^2+29.16}\right)*\frac{b^2+29.16}{2b}$$

Now graph the right side and the left side (y=12.4575), seeing where they cross. Intercept at b=2.6495mm.



Section 8.1

35. First let's take the solution values from problem 1.

$$\begin{array}{ll} \alpha = 73° & a = 41\,ft \\ \beta = 28° & b = 20\,ft \\ \gamma = 79° & c = 42\,ft \end{array}$$

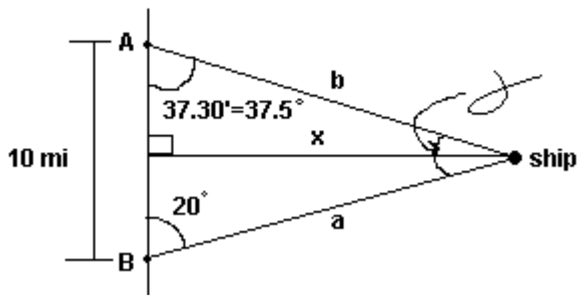
And plug into the equation

$$(a - b)\cos\frac{\gamma}{2} = c\sin\frac{\alpha - \beta}{2}$$

$$(41 - 20)\cos\frac{79}{2} = 42\sin\frac{73 - 28}{2}$$
$$(21)\cos(39.5) = 42\sin(22.5)$$
$$16.2041 = 16.0727$$

*Includes some round-off error.

37.



$$\gamma = 180 - 37.5 - 20 = 122.5°$$
$$\frac{\sin 37.5}{a} = \frac{\sin 20}{b} = \frac{\sin 122.5}{10 miles}$$

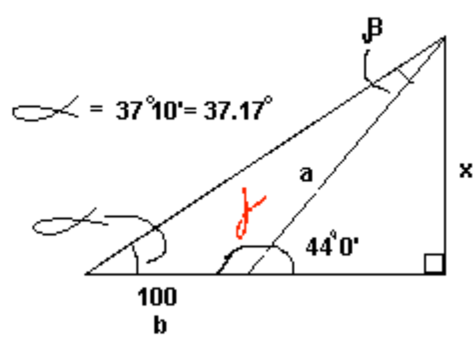
Solve for a

$$a = 10 * \frac{\sin 37.5}{\sin 122.5} = 7.218 miles$$

Now solve for x

$$\sin 20 = \frac{x}{a} = \frac{x}{7.218}$$
$$x = 2.469 miles$$

39.



$\gamma = 180 - 44 = 136°$

$\beta = 180 - 136 - 37.17 = 6.83°$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\frac{\sin 37.17}{a} = \frac{\sin 6.83}{100}$$

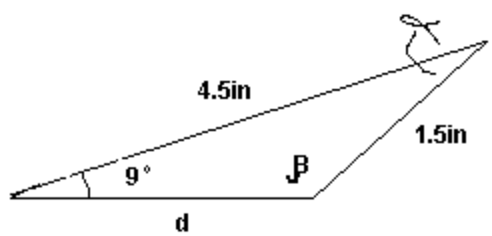
$a = 508.041 ft$

Now solve for the *x* value.

$$\sin 44 = \frac{x}{a} = \frac{x}{508.041}$$

$a = 352.915 \cong 353 ft$

41.



SSA Case

$$\frac{\sin 9^\circ}{1.5} = \frac{\sin\alpha}{d} = \frac{\sin\beta}{4.5}$$

$$\sin\beta = \left(\frac{\sin 9^\circ}{1.5}\right)4.5 = 0.469303$$

$$\beta_1 = \sin^{-1} 0.469303 = 27.9891^\circ$$

$$\beta_2 = 180 - \beta_1 = 152.011^\circ$$

$$\alpha = 180 - 9 - \beta$$

$$\alpha_1 = 143.011^\circ$$

$$\alpha_2 = 18.9891^\circ$$

$$d = \frac{\sin\alpha}{\sin 9} * 1.5$$

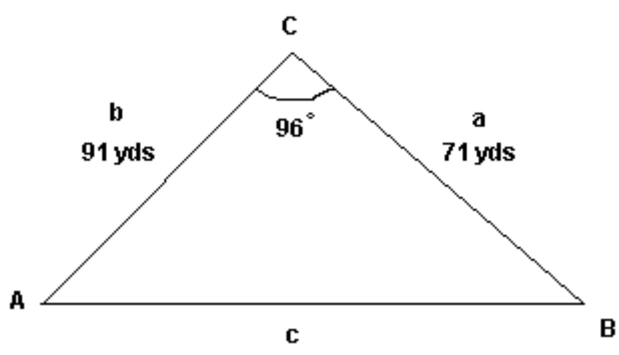
$$d_1 = \frac{\sin 143.011}{\sin 9} * 1.5 = 5.769 \cong 5.8 in$$

$$d_2 = \frac{\sin 18.9891}{\sin 9} * 1.5 = 3.12 in$$

Section 8.2

27. $c^2 = a^2 + b^2 - 2ab\cos\alpha$
$c^2 = a^2 + b^2 - 2ab\cos 90^\circ$
$c^2 = a^2 + b^2 - 0 = a^2 + b^2$

33.



$$c^2 = a^2 + b^2 - 2ab\cos 96^\circ$$

$$c^2 = 71^2 + 91^2 - 2(71)(91)\cos 96^\circ$$

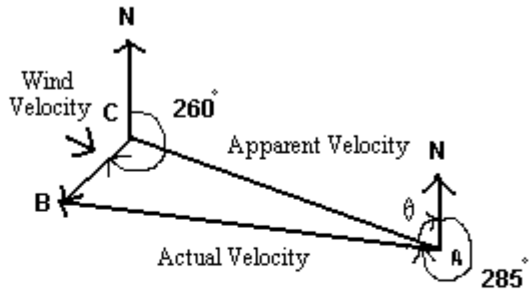
$$c^2 = 14672.7$$

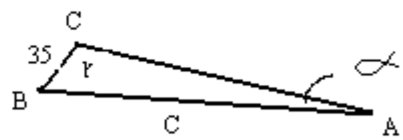
$$c = 121.131 yds \cong 120 yds$$

Use an approximate since we are assuming these are rough measurements.

<u>Section 8.3</u>

25. Draw the figures.





Now use law of cosines.

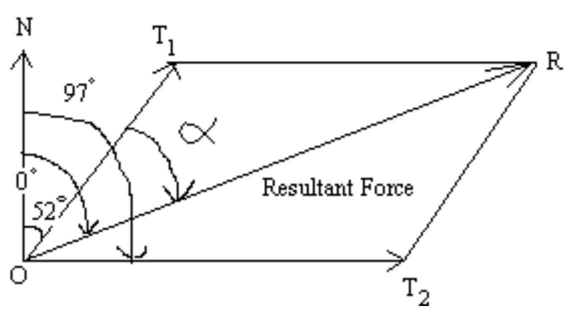
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$c^2 = 35^2 + 230^2 - 2(35)(230)\cos(155°)$$
$$c^2 = 35^2 + 230^2 - 2(35)(230)\cos(155°)$$
$$c = 262.12 mph$$

Now use the law of sines to get the angle.

$$\frac{\sin 155°}{262.12} = \frac{\sin\beta}{35}$$
$$\sin\beta = 0.056431$$
$$\beta = 3.23°$$

Heading = 285-3.23=281.77 degrees

29. Look at the figure and what you are given.



Use the law of cosines to find the magnitude of the resultant force, OR

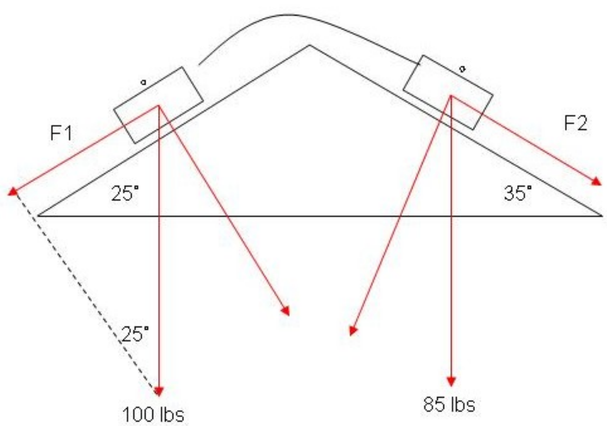
$$OR^2 = OT_1{}^2 + T_1R^2 - 2(OT_1)(T_1R)\cos(\angle OT_1R)$$
$$= (2300)^2 + (1900)^2 - 2(2300)(1900)\cos 135^\circ$$
$$= 15{,}080{,}133\ldots$$
$$OR = \sqrt{15080133...}$$
$$= 3{,}900 \text{ pounds}$$

We use the law of sines to find α

$$\frac{\sin\alpha}{T_1R} = \frac{\sin(OT_1R)}{OR}$$
$$\frac{\sin\alpha}{1900} = \frac{\sin 135^\circ}{3900}$$
$$\sin\alpha = \frac{1900}{3900}\sin 135^\circ$$
$$\alpha = \sin^{-1}\left(\frac{1900}{3900}\sin 135^\circ\right)$$
$$= 20^\circ$$

Then the compass direction $\theta = 52^\circ + \alpha = 72^\circ$

33.



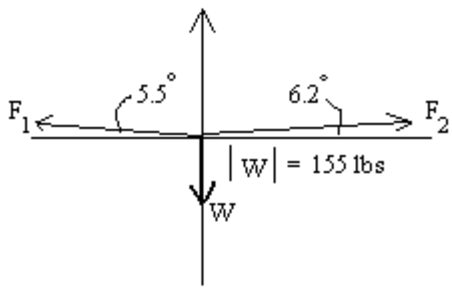
Like figure 10 on page 600, the figure above shows the blocks broken into the respective forces. We are solving for the forces parallel to the ramp. The side with the most downward force (parallel to the ramp) will be the direction of travel.

$$F_1 = 110\sin 25^\circ \qquad F_2 = 85\sin 35^\circ$$
$$= 46.49 lbs \qquad = 48.75 lbs$$

Since F_1 is less then F_2, they will slide to the right.

Section 8.4

45.



F_1 = the tension in the left rope
F_2 = the tension in the right rope

Write each force vector in terms of i and j unit vectors

$$F_1 = |F_1|(-\cos 5.5^\circ)i + |F_1|(\sin 5.5^\circ)j$$
$$F_2 = |F_2|(\cos 6.2^\circ)i + |F_2|(\sin 6.2^\circ)j$$
$$W = -155j$$

For the system to be in static equilibrium, we must have

$$F_1 + F_2 + W = 0$$

which becomes, on addition,

$$[-|F_1|(\cos 5.5^\circ) + |F_2|(\cos 6.2^\circ)]i + [|F_1|(\sin 5.5^\circ) + |F_2|(\sin 6.2^\circ) - 155]j$$
$$= 0i + 0j$$

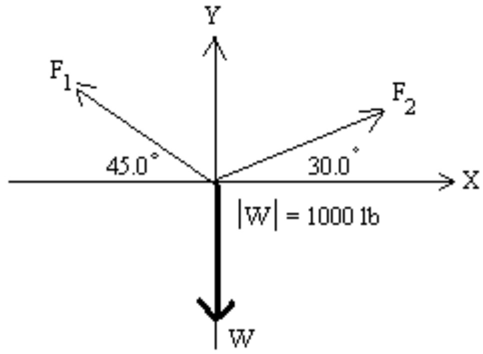
Vectors are equal if and only if their corresponding components are equal.
Therefore:

$$-|F_1|\cos 5.5^\circ + |F_2|\cos 6.2^\circ = 0$$
$$|F_1|\sin 5.5^\circ + |F_2|\sin 6.2^\circ - 155 = 0$$

Solving by standard methods,

$$|F_1| = \frac{155}{\sin 5.5^\circ + \cos 5.5^\circ \tan 6.2^\circ} = 760 lbs \text{ to the left}$$
$$|F_2| = 760\frac{\cos 5.5^\circ}{\cos 6.2^\circ} = 761 lbs \text{ to the right}$$

47. Draw a force diagram with all force vectors in standard position from the origin



F_1 = the tension in left cable
F_2 = the tension in right cable

$$F_1 = |F_1|(-\cos 45.0^\circ)i + |F_1|(\sin 45.0^\circ)j$$
$$F_2 = |F_2|(\cos 30.0^\circ)i + |F_2|(\sin 30.0^\circ)j$$
W = -1000j

For the system to be in static equilibrium we must have,

$$F_1 + F_2 + W = 0$$

which becomes, on addition,

$$[-|F_1|(\cos 45.0^\circ)i + |F_2|(\cos 30.0^\circ)]i + [|F_1|(\sin 45.0^\circ) + |F_2|(\sin 30.0^\circ) - 1000]j = 0i + 0j$$

Since two vectors are equal if and only if their corresponding components are equal:

$$-|F_1|\cos 45.0^\circ + |F_2|\cos 30.0^\circ = 0$$
$$|F_1|\sin 45.0^\circ + |F_2|\sin 30.0^\circ - 1000 = 0$$

solving by standard methods:

$$|F_1| = \frac{1000}{\sin 45.0^\circ + \cos 45.0^\circ \tan 30.0^\circ} = 897lbs \text{ to the left}$$
$$|F_2| = 897\frac{\cos 45.0^\circ}{\cos 30.0^\circ} = 732lbs \text{ to the right}$$

49. $|c| = 400lbs$

$$\cos ABC = \frac{1}{2},\ ABC = 60^\circ$$

therefore:

$$1a = |a|i$$

$$b = |b| - \cos(60.0^\circ)i + |b|\sin 60.0^\circ\, j = -\frac{1}{2}|b|i + \frac{\sqrt{3}}{2}|b|j$$
$$c = -400j$$

for static equilibrium: a + b + c = 0

therefore: $[|a| - \frac{1}{2}|b|]i + [\frac{\sqrt{3}}{2}|b| - 400]j = 0i = 0j$

Since two vectors are equal if and only if their corresponding components are equal:

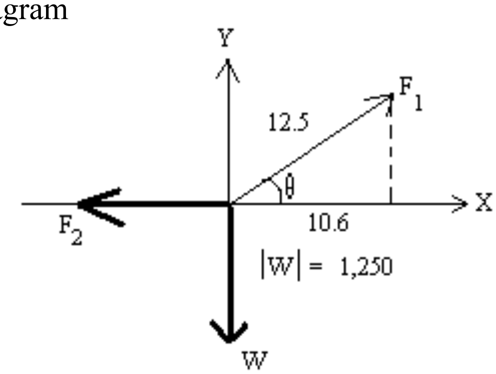
$$|a| - \frac{1}{2}|b| = 0$$
$$\frac{\sqrt{3}}{2}|b| - 400 = 0$$

solving:

$|b| = \frac{2}{\sqrt{3}}(400) = 462lbs$ this corresponds to a tension force of 462 lbs in member CB

$|a| = \frac{1}{2}|b| = 231lbs$ this corresponds to a compression force of 231 lbs in member AB

51. Form a force diagram



F_1 = the force on the horizontal member BC
F_2 = the force on the supporting member AB
W = the downward force (1,250 lbs)

We note: $\cos\theta = \dfrac{10.6}{12.5}$, $\quad \theta = \cos^{-1}\dfrac{10.6}{12.5} = 32.0^\circ$

$$F_1 = -|F_1|i$$
$$F_2 = |F_2|(\cos 32.0^\circ)i + |F_2|(\sin 32.0^\circ)j$$
W = -1,250 j

For the system to be in static equilibrium we must have,

$$F_1 + F_2 + W = 0$$

which becomes:

$$[-|F_1| + |F_2|(\cos 32.0^\circ)]i + [|F_2|(\sin 32.0^\circ) - 1{,}250]j = 0i + 0j$$

Since two vectors are equal if and only if their corresponding components are equal:

$$-|F_1| + |F_2|(\cos 32.0^\circ) = 0$$
$$|F_2|(\sin 32.0^\circ) - 1250 = 0$$

Solving:

$$|F_2| = \frac{1250}{\sin 32.0^\circ} = 2360 lbs$$
$$|F_1| = |F_2|\cos 32.0^\circ = 2000 lbs$$

the force in the member AB is directed oppositely to the diagram---a compression of 2,360 lbs. The force in the member BC is also directed oppositely to the diagram--- a tension of 2000 lbs.

<u>Section 8.5</u>

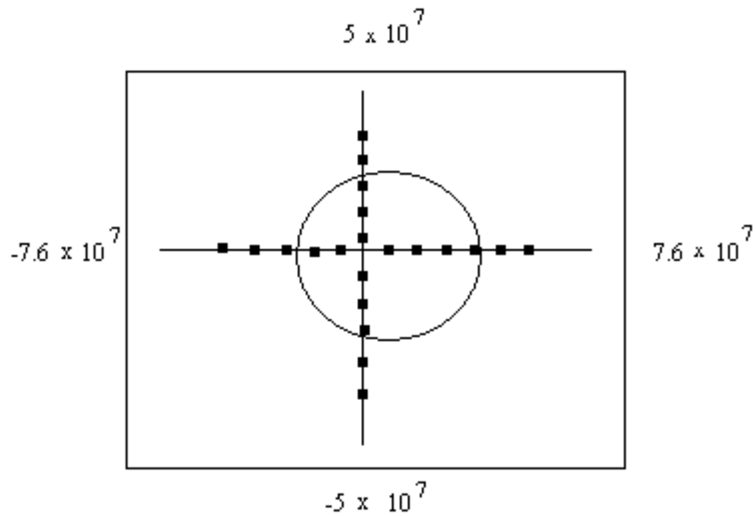
63. $d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

$(r_1, \theta_1) = (4, \frac{\pi}{4}) \qquad (r_2, \theta_2) = (1, \frac{\pi}{2})$

$$d = \sqrt{4^2 + 1^2 - 2(4)(1)\cos(\frac{\pi}{2} - \frac{\pi}{4})}$$
$$d = \sqrt{17 - 8\cos\frac{\pi}{4}}$$
$$d = 3.368 \text{ units}$$

65. 6 knots at 30.0°, 13 knots at 75°, 12 knots at 135°, 9 knots at 180°

69. (A)

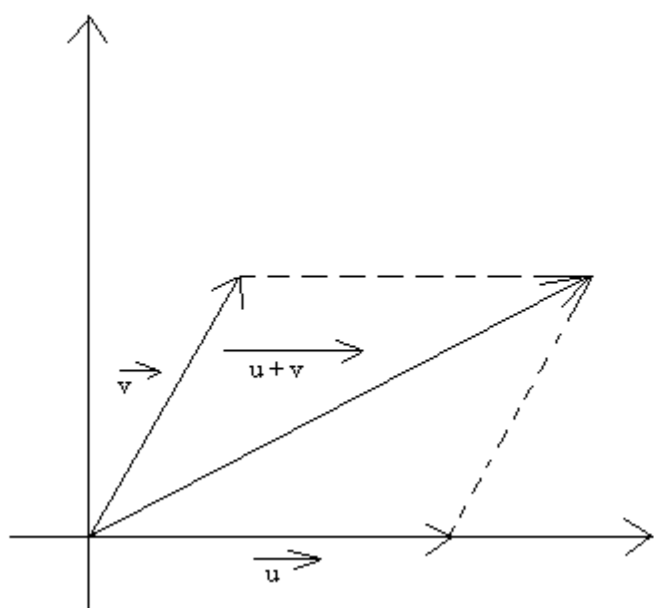


At aphelion, the distance is 4.34 x10^7 mi; at perihelion it is 2.85 x 10^7 mi

(B) Faster at perihelion. Since the distance from the sun to Mercury is less at perihelion than at aphelion, the planet must move faster near perihelion in order for the line joining Mercury to the sun to sweep out equal areas in equal intervals of time.

Section 8.6

35.



$\vec{u}: 20e^{0^\circ i}$

$\vec{v}: 10e^{60^\circ i}$

(A) $\vec{u}: 20e^{0^\circ i} = 20(\cos 0^\circ + i\sin 0^\circ)$
$= 20(1+0i) = 20 + 0i$

$\vec{v}: 10e^{60^\circ i} = 10(\cos 60^\circ + i\sin 60^\circ) = 10(\frac{1}{2} + i\frac{\sqrt{3}}{2}) = 5 + 5i\sqrt{3}$

$(20+0i) + (5+5i\sqrt{3}) = 25 + 5i\sqrt{3}$

(B) $r = \sqrt{(25)^2 + (5\sqrt{3})^2} = \sqrt{625+75} = \sqrt{700} = 26.5$

$\tan\theta = \frac{5\sqrt{3}}{25} = \frac{\sqrt{3}}{5}$ $\qquad \theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{3}}{5} = 19.1^\circ$

$25 + 5i\sqrt{3} = 26.5e^{19.1^\circ i}$

(C) $26.5e^{19.1^\circ i}$ represents a force of 26.5 pounds at an angle of 19.1 degrees with the positive x axis.

<u>Section 8.7</u>

33. $x^3 - 27 = 0$
Thus we require the three cube roots of 27

Write 27 in polar form
$27 = 27e^{0^\circ i}$

use nth root theorem
$27^{1/3} e^{\left(0^\circ/3 + k\cdot 360^\circ/3\right)i} = 3e^{k\cdot 120^\circ i}$ k = 0, 1, 2
thus

$w_1 = 3e^{0^\circ i} = 3\left(\cos 0^\circ + i\sin 0^\circ\right) = 3$

$w_2 = 3e^{120^\circ i} = 3\left(\cos 120^\circ + i\sin 120^\circ\right) = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$

$w_3 = 3e^{240^\circ i} = 3\left(\cos 240^\circ + i\sin 240^\circ\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$

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