AskDrCallahan Calculus 1

Teacher's Guide

3rd Edition
rev 081712

Dale Callahan, Ph.D., P.E.
Lea Callahan, MSEE, P.E.

Welcome to AskDrCallahan Calculus 1

Start Here!

1. Make sure you have all of the following.

- Calculus by James Stewart, 2nd ed by Brooks/Cole ISBN: 0534377181
- Solutions manual titled "Single Variable Calculus: Concepts and Contexts" ISBN: 0534379230
- Teacher's Guide for Calculus 1 (This document on CD)
- Graphing Calculator
- DVD set of seven (7) DVDs

2. Both the teacher and the student should put in the first DVD and play the course introduction.

3. Review the syllabus. Perhaps make a copy of the syllabus and add some dates to help you plan. The syllabus is designed like most college courses, so using it will be excellent preparation for what is to come. (The syllabus can also be downloaded from the website under support/downloads.)

4. Have your students begin working in chapter 1.Note there are no videos for chapter 1 since chapter 1 is an introduction and will be a review for many students. Using the syllabus as a guide, allow the student to move at a comfortable pace making sure they understand the material.

5. Once chapter 1 has been completed, start the student with the first video for chapter 2. For help contact us at support@askdrcallahan.com or see our Homework Help page at www.askdrcallahan.com.

Courses by AskDrCallahan

- Algebra 1
- Geometry
- Algebra II with trigonometry (Can be sold as Algebra II or Trig only)
- Calculus 1 (Equivalent to Calculus 1 at most universities)
- Discovering the Entrepreneur in You

See website for more details.

Website: www.askdrcallahan.com

Email: support@askdrcallahan.com

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How to run the course

The best way to manage the course is for students to take one section of the text at a time and work through it in a logical fashion. We recommend watching one section of the DVD then working the problems in the corresponding section of the text. Go back to the DVD and text examples as needed to make sure most of the problems can be easily worked and understood.

Before moving on to the next section, make sure the current section is understood. Be aware some sections are more complex than others – so things will vary.

If questions arise that are not answered – please email us. We will provide answers or other help as needed – including emailing solutions, video solutions, or helping on the phone.

Pace

The syllabus has the projected dates we would use if we taught this material in a classroom meeting one day per week for two hours at a time. However, even in the classroom we will not always be on schedule since adjustments are always being made. You should not be overly concerned with following the schedule exactly – but use it as a guide. If you need to slow down – even significantly – to make sure the concepts are understood you are doing the right thing.

This is college level material. The only difference is pace - a college course would typically do Calculus 1 in one semester.

Note the difficulty of calculus is usually not calculus itself, but instead algebra. Students who do not have a strong handle on algebra will have similar struggles here as they have had in other algebra courses. Stick with it. Review the algebra as needed and adjust the pace if required. Most students who struggle with calculus in college struggle because weak skills in the fundamentals – usually basic algebra. Time spent here will be very valuable.

Suggested problem set

The syllabus lists suggested problems. Like the schedule, these problems are a suggested guide - what you might expect to see in a college course. Work more or less as needed – however avoid the temptation (or negotiations) to skip most of the problems in a section. We have carefully chosen problems that need to be understood. It is RARE that someone can just look over problems and say they know how to work them after looking at examples. Math is like music – it must be practiced to become proficient.

Also resist the temptation (some parents have) to assign ALL of the problems or all of the odd problems. While it is possible for a student to do all of the problems, the amount of time needed would be significant and likely impact other courses.

Preview and Chapter 1

These sections of the text are not on the video because they are mostly review and course preview. We do encourage students to go through these sections and work some problems. While these sections are review for many students, others may have never made it quite this far in their previous work.

We cover many of the same ideas in the videos as we go through the sections. The main point we want students to understand is calculus has two key concepts. First, finding the slope of any point in any curve. Second is finding the area under any curve.

Chapter 2 (Limits)

Chapter 2 focuses on the concept of a limit to find the slope. This concept is key to the definition of the derivative. We encourage the student to use graphs (with calculators) and sketches as much as possible to get a visual feel for the material – we have found that to be very helpful in the understanding.

Chapter 3 (Derivatives)

Derivatives it the key concept for Calculus 1, and all of the rules of derivatives are covered in chapter 3. After doing limits in chapter 2, chapter 3 should be easier since derivatives are a fast tool for doing limits.

Chapter 4 (Applications of Derivatives)

A majority of chapter 4 is focused on the applications of the tools covered in chapters 2 and 3. These applications in include physics, business, economics, biology, medicine and others. These problems are often word problems – because in reality to apply math we have to start with a written description of the problem. This will require the student to sketch out the situation. Often the key step is the understanding of the basic geometrical relationships described in the problem.

Syllabus for Calculus 1

Textbooks and Calculators
Required
1. *Calculus: Concepts and Contexts, 2nd Edition, James Stewart Brooks/Cole Publishing, ©2001, ISBN: 0534377181.*

2. *Graphing Calculator such as a TI-84 Plus, TI-86, or TI-89. [Calculators with built in CAS (Computer Algebra System Calculator) such as a TI-89, HP 40G, Casio FX 2.0 are often not allowed on the ACT.]*

Optional
1. *Student Solutions Manual for Stewart's Single Variable Calculus: Concepts and Contexts (with CD), 2nd Edition, Jeffery A. Cole Anoka-Ramsey Community College Published by Brooks/Cole, © 2001, ISBN: 0-534-37923-0.*

2. *How to Ace Calculus: The Streetwise Guide by Colin Adams, Joel Hass and Abigail Thompson*

Notes:

- This course is equivalent to a Calculus I course at most universities. It is also most of the material needed for the AP Calculus AB exam. To receive AP college credit for this course you will need to contact the university you plan to attend.
- Chapter 1 of the text is mostly a review of pre-calculus. The videos do NOT cover this section, but the student is encouraged to work through ht problems as a refresher.
- Parents may wish to shorten the time so this semester's material will be covered in one semester as opposed to one year. Calculus II is offered for students who are planning to take the AP Calculus BC exam or are planning to go into math intensive study - such as the natural sciences or engineering.

Suggested Grading
75% Three tests (25% each)
25% Final Exam

Suggested Schedule

Week	Sect	Title	Suggested HW Problems
1	1.1	Four Ways to Represent a Function	1, 5, 7, 19, 21, 23, 27, 31, 35, 39, 41, 43-50, 53
1	1.2	Mathematical Models	1, 3, 5, 7, 9, 11, 13
2	1.3	New Functions from Old Functions	1, 3, 5, 7, 15, 17, 21, 23, 25, 31, 37, 38, 49, 51, 53, 57
2	1.4	Graphing Calculators and Computers	1, 3, 7, 15, 17, 23
3	1.5	Exponential Functions	7-13 odd, 15, 23
3	1.6	Inverse Functions and Logarithms	1, 5-11 odd, 17, 23, 27, 31, 35, 37, 39, 49, 51, 57
4	1.7	Parametric Curves	1-13 odd, 27, 33
4	2.1	The Tangent and Velocity Problems	1, 3, 5, 7
5	2.2	The Limit of a Function	1, 4, 9, 11, 13
6		TEST 1 (Covers 1.1 to 2.2)	
7	2.3	Calculating Limits using the Limit Laws	1, 3, 5, 9, 15, 17, 25, 27, 33, 35, 39, 41
7	2.4	Continuity	1, 3, 5, 9, 11, 13, 15, 17, 19, 23, 25, 29, 31, 33, 35, 37
8	2.5	Limits Involving Infinity	1, 3, 5, 7, 9, 13, 15, 19, 23, 25, 27, 35, 39, 41, 43
8	2.6	Tangents, Velocities, and Other Rates of Change	1-19 odd
9	2.7	Derivatives	1-9 odd, 13, 17, 19, 21, 23, 25, 29, 31, 35, 36
9	2.8	The Derivative as a Function	1, 3, 5, 9, 11, 15, 19, 23, 25, 31, 35, 43, 45
10	2.9	Linear Approximations	1, 3, 5, 9, 13
10	2.10	What Does f' Say about f?	1, 3, 5, 9, 12, 17, 19, 21, 25, 27
11	3.1	Derivatives of Polynomials and Exponential Functions	1, 5-21 odd, 31, 37, 41, 43, 45, 49, 53, 55, 59
11	3.2	The Product and Quotient Rules	1-13 odd, 17, 27-39 odd
12	3.3	Rates of Change in the Natural and Social Sciences *(mostly optional)*	1-13 odd, 19
12	3.4	Derivatives of Trigonometric	

		Functions	1-11 odd, 13, 15, 17, 23-33 odd, 37, 41
13		TEST 2 (Covers 2.3 to 3.4)	
14	3.5	The Chain Rule	1, 3, 5, 7, 11, 15, 17, 19, 21, 27, 29, 37, 41, 45, 47, 49, 53, 65, 67, 73
14	3.6	Implicit Differentiation	1, 3, 5, 11, 13, 15, 17, 23, 25, 27, 31, 37, 41, 43, 45, 49, 51
15	3.7	Derivatives of Logarithmic Functions	3, 5, 9, 11, 15, 17, 19, 22, 27, 29, 31, 35, 39
15	3.8	*Linear Approximations and Differentials (This section can be skipped and is skipped on the videos)*	*Optional: 1, 3, 5, 7, 17, 19, 21*
16	4.1	Related Rates	1, 5, 7, 11, 13, 14, 17, 19, 21, 23, 29, 33
16	4.2	Maximum and Minimum Values	1, 3, 5, 7, 11, 15, 17, 19, 25, 27, 29, 33-43 odd, 51, 55
17	4.3	Derivatives and the Shapes of Curves	1, 3, 5, 11, 13, 17, 21, 23, 25, 27, 29, 41, 45
17	4.4	Graphing with Calculus and Calculators	1, 5, 11, 19
18	4.5	Indeterminate Forms and L'Hospital's Rule	1, 3, 5, 7, 11, 15, 23, 29, 33, 41, 43, 51, 53, 57
18	4.6	Optimization Problems	7, 9, 11, 13, 17, 21, 23, 27, 29, 33, 39
19	4.7	Applications to Economics	*1, 3, 9, 15, 17*
19	*4.8*	*Newton's Method (This section can be skipped and is skipped on the videos)*	*(Optional)*
20	4.9	Antiderivatives	1, 5, 7, 9, 11, 13, 15, 19, 23, 25, 29, 35, 43, 45
21		TEST 3 (Covers 3.5 to 4.9)	
22		Review for final	
23		Final Exam (Covers Ch 1 to Ch 4)	

Calculus Test Grading Guide

Welcome teacher!

This test-grading guide is designed to make the grading of tests as easy as possible while at the same time encouraging learning by the student.

When to take the tests

The tests should be taken after the student has completed the sections covered by the test – as laid out on the syllabus. The syllabus indicates how we would deliver the tests in a classroom environment, but you can give the test whenever the student is ready.

How to take the tests

The tests problems may come from the textbook or other sources, but problems should be very similar to the homework. The answers to the test are NOT in the textbook, you have the answers and the solutions in this grading guide only. It is recommended that this test be taken open book and open notes. In addition, you might find it best to allow the student to work the test over a few days. IN a college environment the students would have between 1 to 2 hours to take these tests.

How to grade

You will find the sheets used to grade the test following these notes. We recommend you grade CORRECT ANSWER ONLY. We also recommend two (or more) tries for problems the student misses on the test.

Here is how we do it. (See the attached example)

First – we deliver the test, then grade for correct answer only. We give the student the grade with problems marked correct or incorrect. The initial grades may be low, but we encourage the student not worry about this yet.

Second – we allow the student to go back and attempt to correct the problems they missed. This method encourages them to learn from their mistakes. We then re-grade the problems they initially had wrong, giving partial credit for the accurate solutions.

We have included an example grading sheet showing a student who got 10 of 12 problems correct on the first try. Then on the retest they got the other two problems correct. We graded as giving them 50% of the original credit and adding it to the final grade.

The solution guide has the solution to the problem with a box drawn around the final answer - as you should expect the student to have done. Note that in some problems the student is asked to verify the solution. In this case you just need to make sure the steps the student took are similar to what is found in the solution. Sometimes the student may answer YES or NO since they are being asked to verify if the solution is true or not.

Adjustments you can make

You may want to allow the student to try a third or fourth time. This is not cheating – the goal is to learn!

You might also want to adjust the partial credit on the rework. To adjust, use another number on line e of the grade sheet. (Using 80 instead of 50 would give 80% of the points for corrected problems.)

Filing and grade management

We know that each person has different filing requirements, so if you choose to not keep the grades in this solutions book feel free to copy the grade sheets for easier filing. The grading sheets are also available on the website under support/downloads.

Test Grade Sheet

Student ________EXAMPLE____

Course _____Calculus__________

Test Number ______1___________

Attempt # 1

a. Number of problems correct ___8___

b. Total number of problems ____10____

c. Grade (100*a/b) ____80____(round up to nearest integer)

Attempt #2 d. Number of problems fixed ____1____

e. Points added (50*d/b) ____5____

Test Grade

f. Final Grade (c + e) ____85____(round up to nearest integer)

Test Grade Sheet

Student ______________________

Course _______________________

Test Number _________________

Attempt # 1

a. Number of problems correct _______

b. Total number of problems ________

c. Grade (100*a/b) ________(round up to nearest integer)

Attempt #2 d. Number of problems fixed ________

e. Points added (50*d/b) ________

Test Grade

f. Final Grade (c + e) ________(round up to nearest integer)

Test Grade Sheet

Student ______________________

Course _______________________

Test Number _________________

Attempt # 1

a. Number of problems correct _______

b. Total number of problems ________

c. Grade (100*a/b) ________(round up to nearest integer)

Attempt #2 d. Number of problems fixed ________

e. Points added (50*d/b) ________

Test Grade

f. Final Grade (c + e) ________(round up to nearest integer)

Test Grade Sheet

Student ________________________

Course _________________________

Test Number __________________

Attempt # 1

a. Number of problems correct _______

b. Total number of problems ________

c. Grade (100*a/b) ________(round up to nearest integer)

Attempt #2 d. Number of problems fixed ________

e. Points added (50*d/b) ________

Test Grade

f. Final Grade (c + e) ________(round up to nearest integer)

Test Grade Sheet

Student ______________________

Course _______________________

Test Number _________________

Attempt # 1

a. **Number of problems correct _______**

b. **Total number of problems ________**

c. **Grade (100*a/b) ________(round up to nearest integer)**

Attempt #2

d. **Number of problems fixed ________**

e. **Points added (50*d/b) ________**

Test Grade

f. **Final Grade (c + e) ________(round up to nearest integer)**

Name: ____________________

CALCULUS 1
Test 1
Sections 1.1 to 2.2

JUSTIFY YOUR ANSWERS!

1. (Ch 1.2) The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May it cost her \$380 to drive 480 miles and in June it cost her \$460 to drive 800 miles.

(a) Express the monthly cost C as a function of the distance driven d, assuming that a linear relationship gives a suitable model.

(b) Use part (a) to predict the cost of driving 1500 miles per month.

(c) Draw the graph of the linear function. What does the slope represent?

(d) What does the y-intercept represent?

(e) Why does a linear function give a suitable model in this situation?

2. (Ch 1.3) Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains given

$f(x) = 1 - 3x$, $g(x) = 5x^2 + 3x + 2$.

3. (Ch 1.5) Make a rough sketch of the graph of the function $y = 2 + 5(1 - e^{-x})$.

4. (Ch 1.6) Solve for x.

(a) $e^{2x+3} - 7 = 0$.

(b) $\ln(5-2x) = -3$

5. (Ch 1.7) Sketch the curve by using parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

$x = 2\cos t$, $y = t - \cos t$, $0 \le t \le 5$

6. (Ch 2.1) The position of a car is given by the values in the table.

t (seconds)	0	1	2	3	4	5
s (feet)	0	10	32	70	119	178

(a) Find the average velocity for the time period beginning when t=2 and lasting

i. 3 s   ii. 2 s   iii. 1 s

(b) Use the graph of s as a function of t to estimate the instantaneous velocity when t=2.

7. (Ch 2.2) Sketch the graph of the following function and use it to determine the values of a for which $\lim_{x \to a} f(x)$ exists.

$$f(x) = \begin{cases} 2 - x & \text{if} \quad x < -1 \\ x & \text{if} \quad -1 \le x \le 1 \\ (x-1)^2 & \text{if} \quad x \ge 1 \end{cases}$$

8. (Ch 2.2) Sketch the graph of an example of a function f that satisfies all of the given conditions.

$\lim_{x\to 0^-} f(x) = 1$
$\lim_{x\to 0^+} f(x) = -1$
$\lim_{x\to 2^-} f(x) = 0$
$\lim_{x\to 2^+} f(x) = 1$
$f(2) = 1$
$f(0)$ is undefined

Name: ______________________

CALCULUS 1
Test 2
Sections 2.3 to 3.4

JUSTIFY YOUR ANSWERS!

1. (Ch 2.3) Evaluate the limit (if it exists) and justify each step.

(a) $\lim_{x\to 2} \dfrac{2x^2+1}{x^2+6x-4}$

(b) $\lim_{x\to 2} \dfrac{x^4-16}{x-2}$

2. (Ch 2.4) Explain why the function is discontinuous at a=1. Sketch the graph of the function.

$$f(x) = \begin{cases} 1+x^2 & x<1 \\ 4-x & x\geq 1 \end{cases}$$

3. (Ch 2.5) Sketch the graph of an example of a function f that satisfies all of the given conditions. (Pay attention to the + and – signs.)

$\lim_{x\to 0^+} f(x) = \infty$, $\lim_{x\to 0^-} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to -\infty} f(x) = 1$

4. (Ch 2.5) Find the limit of

$$\lim_{x\to 5^-} \frac{e^x}{(x-5)^3}$$

5. (Ch 2.7) Sketch the graph of a function g for which $g(0)=0$, $g'(0)=3$, $g'(1)=0$, and $g'(2)=1$.

6. (Ch 3.1) Find $f'(a)$ for $f(t) = t^4 - 5t$. (Find this using the Limit function definition of derivative and then check with the chapter 3 material.)

7. (Ch 3.1 and 3.2) Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative. (Find this using the Limit function definition of derivative and then check with the chapter 3 material.)

(a) $f(x) = 5 - 4x + 3x^2$

(b) $f(x) = \dfrac{3+x}{1-3x}$

8. (Ch 3.1) Differentiate the function.

(a) $F(x) = -4x^{10}$

(b) $g(x) = 5x^8 - 2x^5 + 6$

(c) $H(s) = (s/2)^5$

9. (Ch 3.2) Differentiate.

(a) $y = \dfrac{e^x}{1+x}$

(b) $f(u) = \dfrac{1-u^2}{1+u^2}$

(c) $f(x) = \dfrac{ax+b}{cx+d}$

10. (Ch 3.4) Differentiate.

(a) $f(x) = x \sin x$

(b) $y = \dfrac{\sin x}{1 + \cos x}$

(c) $y = \dfrac{\tan x - 1}{\sec x}$

11. (Ch 3.4) Prove that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$. (Hint: Start with f'(1/cosx))

Name: ____________________

CALCULUS 1
Test 3
Sections 3.5 to 4.2

JUSTIFY YOUR ANSWERS!
Show all work!

Note: The letters a, b, c, and C all denote constants.

1. (Ch 3.5) Find the derivative of the function.

(c) $F(x) = (x^2 - x + 1)^3$
(d) $f(t) = \sqrt[3]{1 + \tan t}$
(e) $y = a^3 + \cos^3 x$
(f) $y = e^{-5x} \cos 3x$
(g) $y = \sin(\sin(\sin x))$

2. (Ch 3.6) Using $4x^2 - 9y^2 = 36$

(a) Find $\dfrac{dy}{dx}$ (or y') by implicit differentiation.
(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.
(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

3. (Ch 3.6) Find dy/dx by implicit differentiation for

$$x^2 - 2xy + y^3 = C.$$

4. (Ch 3.7) Differentiate the function.

(a) $f(x) = \ln(x^2 + 10)$
(b) $f(x) = \cos(\ln x)$
(c) $f(t) = \dfrac{1 + \ln t}{1 - \ln t}$

5. (Ch 4.1) (a) If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.

   (b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1m/s, how fast is the area of the spill increasing when the radius is 30m?

6. (Ch 4.1) A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point (2,3), the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

7. (Ch 4.2) Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f.

(a) $f(x) = 3 - 2x, \;\; x \leq 5$

(b) $f(x) = e^x$

Name: ____________________

CALCULUS 1
Final Exam (Test 4)
Sections 1.1 to 4.9

JUSTIFY YOUR ANSWERS!
Show all work!

1. (Ch 2) Evaluate the following limits.

   (h) $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$

   (i) $\lim_{x \to \infty} \frac{x^2 + x}{3 - x}$

2. (Ch 2) Find the equation of the tangent line to the graph of $y = f(x) = \frac{\ln x}{x}$ at the point (1,0).

3. (Ch 3) Differentiate $3e^{2x+3} + \sqrt{x} \ln x$.

4. (Ch 3) Differentiate $\ln(\cos(2x))$.

5. (Ch 4) At noon, two ships start moving from the same point. Ship A is sailing north at 15mi/h and ship B is sailing east at 20 mi/h. How fast is the distance between the ships changing at 3:00 P.M.?

6. (Ch 4) Consider the function $f(x) = x^4 - 2x^2 + 3$.

   (a) Where is the function increasing and where is the function decreasing? Write down your answers interval notation.

   (b) What are the local Maxima and minima of *f(x)*?

   (c) Where is the *f(x)* concave up and where is the *f(x)* concave down? Write down your answers in interval notation.

7. (Ch 4) A rectangle storage container with a closed top is to have a volume of 72 ft^3. The length of its base is twice the width. Find the dimensions of the box that minimize the amount of material used.

8. (Ch 4) Find $g(x)$ when $g''(x) = 6x^2 + e^x$, $g(0) = c^2$, and $g'(0) = 5$.

Name: **Solution**

CALCULUS 1
Test 1
Sections 1.1 to 2.2

JUSTIFY YOUR ANSWERS!

9. (Ch 1.2) The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May it cost her \$380 to drive 480 miles and in June it cost her \$460 to drive 800 miles.

(f) Express the monthly cost C as a function of the distance driven d, assuming that a linear relationship gives a suitable model.

Answer

$$m = slope = \frac{460 - 380}{800 - 480} = 0.25 \ \textit{dollars per mile}$$

Note we have two points, (800, 460) and (480, 380).

Now use the equation of a line $C - C1 = m(d - d1)$ *where we are using costs for the y axis and distance for the x axis.*

$C - 460 = 0.25(d - 800)$

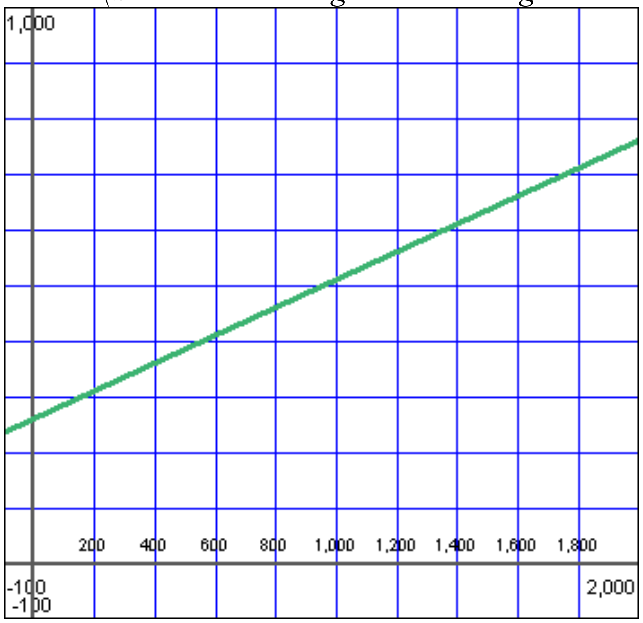
or $C = 0.25d + 260$

(g) Use part (a) to predict the cost of driving 1500 miles per month.

Answer: $C = 0.25d + 260 = 0.25 * 1500 + 260 = 375 + 260 = \$635$

(h) Draw the graph of the linear function. What does the slope represent?

Answer (Should be a straight line starting at zero similar to that shown below.)



(i) What does the y-intercept represent?

Answer: Cost not driving the car at all. Note this cost is $260. Implies the fixed costs such as insurance.

(j) Why does a linear function give a suitable model in this situation?

Answer: Cost increases per mile.

10. (Ch 1.3) Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains given

$f(x) = 1 - 3x$, $g(x) = 5x^2 + 3x + 2$.

Answer

$$f \circ g = f[g(x)] = 1 - 3(5x^2 + 3x + 2) = 1 - 15x^2 - 9x - 6 = -15x^2 - 9x - 5$$

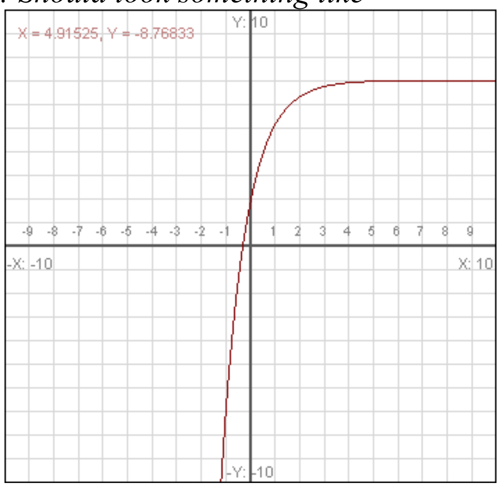
$$g \circ f = g[f(x)] = 5(1-3x)^2 + 3(1-3x) + 2 = 5(1 - 3x - 3x + 9x^2) + 3 - 9x + 2$$
$$= 5 - 15x - 15x + 45x^2 + 3 - 9x + 2 = 45x^2 - 39x + 10$$

$$f \circ f = f[f(x)] = 1 - 3(1 - 3x) = 1 - 3 + 9x = 9x - 2$$

$$g \circ g = g[g(x)] = 5(5x^2 + 3x + 2)^2 + 3(5x^2 + 3x + 2) + 2$$
$$= 5(25x^4 + 15x^3 + 10x^2 + 15x^3 + 9x^2 + 6x + 10x^2 + 6x + 4) + 15x^2 + 9x + 6 + 2$$
$$= 125x^4 + 75x^3 + 50x^2 + 75x^3 + 45x^2 + 30x + 50x^2 + 30x + 20 + 15x^2 + 9x + 6 + 2$$
$$= 125x^4 + 150x^3 + 160x^2 + 69x + 28$$

11. (Ch 1.5) Make a rough sketch of the graph of the function $y = 2 + 5(1 - e^{-x})$.

Answer: Should look something like



12. (Ch 1.6) Solve for x.

(c) $e^{2x+3} - 7 = 0$.

Answer

$$e^{2x+3} = 7$$
$$\ln(e^{2x+3}) = \ln(7)$$
$$2x + 3 = 1.946$$
$$2x = -1.054$$
$$x = -0.527$$

(d) $\ln(5 - 2x) = -3$

Answer

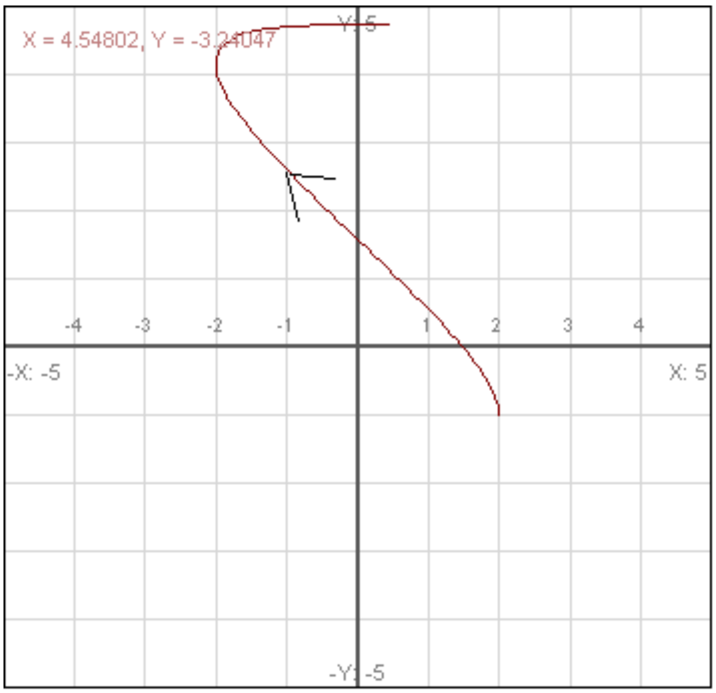
$$\ln(5 - 2x) = -3$$
$$e^{\ln(5-2x)} = e^{-3}$$
$$5 - 2x = e^{-3}$$
$$x = \frac{5 - e^{-3}}{2} = 2.475$$

13. (Ch 1.7) Sketch the curve by using parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

$$x = 2\cos t\,,\quad y = t - \cos t\,,\quad 0 \le t \le 5$$

Answer

Should look similar to below.



14. (Ch 2.1) The position of a car is given by the values in the table.

t (seconds)	0	1	2	3	4	5
s (feet)	0	10	32	70	119	178

(c) Find the average velocity for the time period beginning when t=2 and lasting

i. 3 s ii. 2 s iii. 1 s

(d) Use the graph of s as a function of t to estimate the instantaneous velocity when t=2.

Answer

Part a)

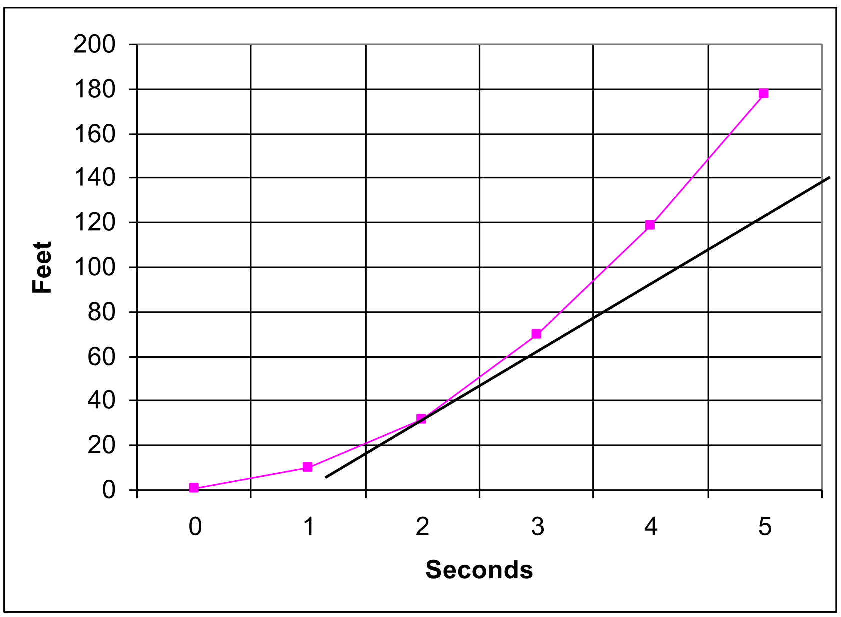
i. $avg = \dfrac{178-32}{5-2} = 48.67\, ft/sec$

ii. $avg = \dfrac{119-32}{4-2} = 43.5\ ft/sec$

iii. $avg = \dfrac{70-32}{3-2} = 38\ ft/sec$

Part b) See below a plot of the data. The heavy black line is an approximate tangent line. Looking at the slope of the tangent line you would get approximately

$$\frac{140-20}{4} = 30\, ft/\sec$$

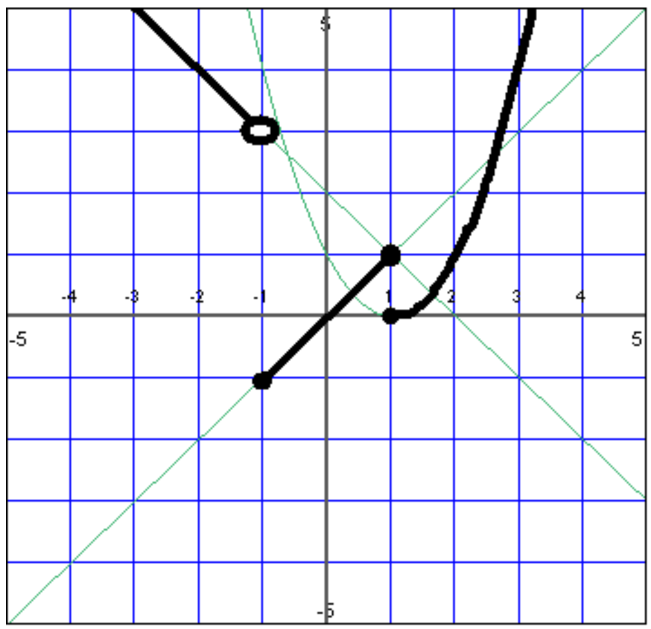


15. (Ch 2.2) Sketch the graph of the following function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

$$f(x) = \begin{cases} 2-x & \text{if} \quad x < -1 \\ x & \text{if} \quad -1 \le x \le 1 \\ (x-1)^2 & \text{if} \quad x \ge 1 \end{cases}$$

Answer

The sketch should look like the dark heavy lines below. Should have three distinct lines. Also the circles at the end of the lines should be solid (closed) or open as indicated in the figure.



The limit exists except where the right and left hand limits are different – which is as x approaches -1 and 1.

16. (Ch 2.2) Sketch the graph of an example of a function f that satisfies all of the given conditions.

$\lim_{x\to 0^-} f(x) = 1$

$\lim_{x\to 0^+} f(x) = -1$

$\lim_{x\to 2^-} f(x) = 0$

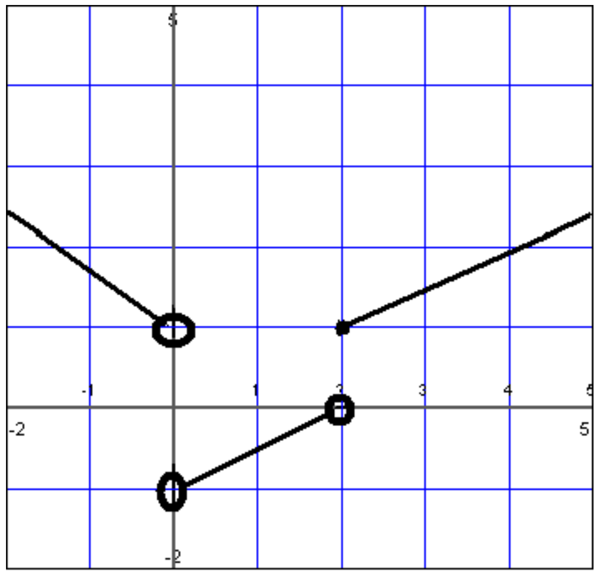
$\lim_{x\to 2^+} f(x) = 1$

$f(2) = 1$

$f(0)$ is undefined

Answer

Should look like the graph below. The end points (black dots) are what is critical. How the line looks between the dots does not matter at all.



Name: **Solution**

CALCULUS 1
Test 2
Sections 2.3 to 3.4

JUSTIFY YOUR ANSWERS!

1. (Ch 2.3) Evaluate the limit (if it exists) and justify each step.

(j) $\lim_{x\to 2} \dfrac{2x^2+1}{x^2+6x-4}$

(k) $\lim_{x\to 2} \dfrac{x^4-16}{x-2}$

Answer

a) $\lim_{x\to 2} \dfrac{2x^2+1}{x^2+6x-4} = \dfrac{2(2)^2+1}{(2)^2+6(2)-4} = \dfrac{9}{12} = \dfrac{3}{4}$

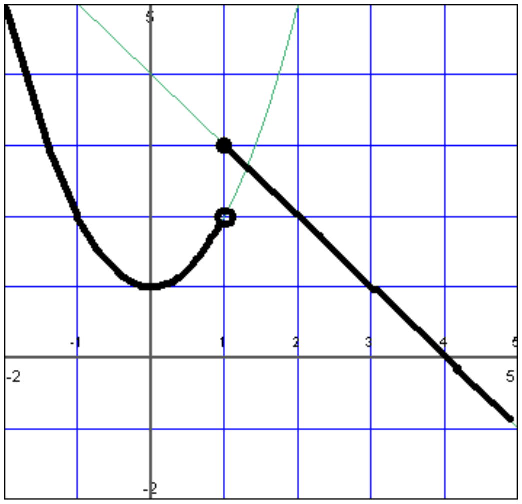
b) $\lim_{x\to 2} \dfrac{x^4-16}{x-2} = \lim_{x\to 2} \dfrac{(x^2-4)(x^2+4)}{x-2} = \lim_{x\to 2} \dfrac{(x-2)(x+2)(x^2+4)}{x-2}$
$= \lim_{x\to 2}(x+2)(x^2+4) = (2+2)(2^2+4) = 4(8) = 32$

2. (Ch 2.5) Explain why the function is discontinuous at a=1. Sketch the graph of the function.

$$f(x) = \begin{cases} 1+x^2 & x<1 \\ 4-x & x\ge 1 \end{cases}$$

Answer

At a=1 the value is 2 for $x<1$ and 3 for $x \ge 1$. Therefore at 1 we have a discontinuity.

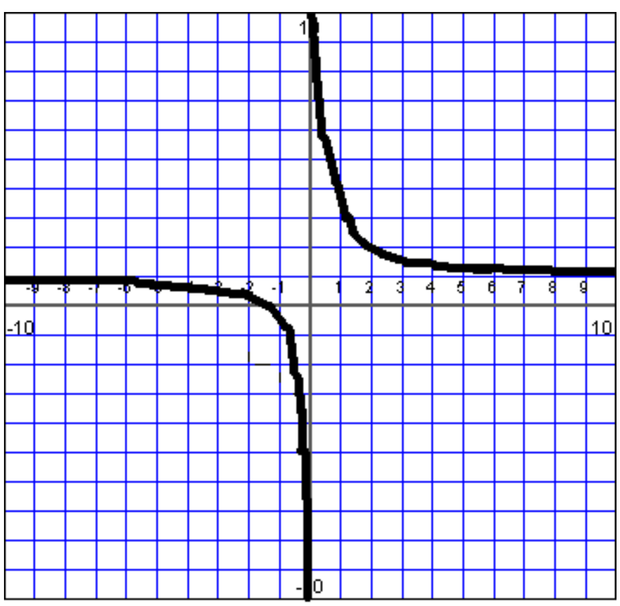


Above is shown a sketch of the graph. The heavy black line is a sketch of what you expect in the solution. The lighter line is part of the original two functions, but not part of the solution.

3. (Ch 2.5) Sketch the graph of an example of a function f that satisfies all of the given conditions. (Pay attention to the + and – signs.)

$\lim_{x\to 0^+} f(x) = \infty$, $\lim_{x\to 0^-} f(x) = -\infty$, $\lim_{x\to \infty} f(x) = 1$, $\lim_{x\to -\infty} f(x) = 1$

Answer – Should look something like below. Critical points are where the graph ends right and left and where it goes up and down. Everything else may be different.



4. (Ch 2.5) Find the limit of

$$\lim_{x\to 5^-} \frac{e^x}{(x-5)^3}$$

Answer

$$\lim_{x\to 5^-} \frac{e^x}{(x-5)^3} = \frac{e^5}{(5^- - 5)^3} = \frac{e^5}{0} = \infty$$

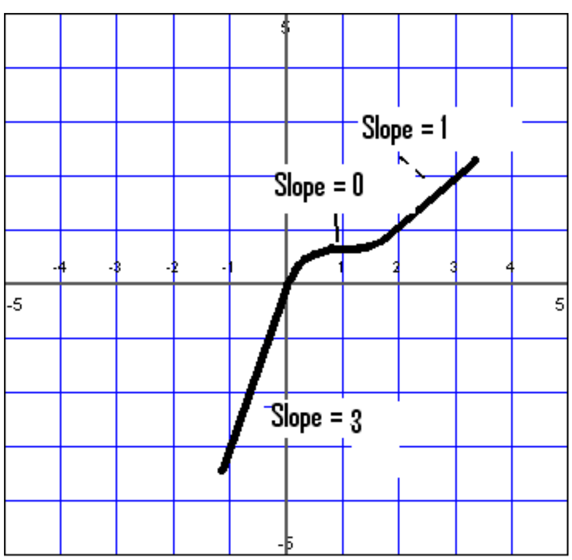
But we must find if negative or positive infinity so try

$$\frac{e^{4.5}}{(4.5-5)^3} = \frac{+}{-}$$

Therefore the answer approaches negative infinity or $-\infty$.

5. (Ch 2.7) Sketch the graph of a function *g* for which *g(0)=0, g'(0)=3, g'(1)=0,* and *g'(2)=1*.

Answer



6. (Ch 3.1) Find *f'(a)* for $f(t) = t^4 - 5t$. (Find this using the Limit function definition of derivative and then using the derivative rules.)

   Answer

   a) Using the Limit Definition of Derivatives

$$\frac{df(t)}{dt} = \underset{h\to 0}{Lim}\frac{f(x+h) - f(x)}{h} = \underset{h\to 0}{Lim}\frac{\left((t+h)^4 - 5(t+h)\right) - \left(t^4 - 5t\right)}{h}$$
$$= \underset{h\to 0}{Lim}\frac{(t+h)^4 - 5t - 5h - t^4 + 5t}{h} = \underset{h\to 0}{Lim}\frac{(t+h)^4 - 5h - t^4}{h}$$
$$= \underset{h\to 0}{Lim}\frac{t^4 + 4t^3h + 5t^2h^2 + 4h^3t + h^4 - 5h - t^4}{h}$$
$$= \underset{h\to 0}{Lim}\frac{4t^3h + 5t^2h^2 + 4h^3t + h^4 - 5h}{h}$$
$$= \underset{h\to 0}{Lim}\frac{h\left(4t^3 + 5t^2h + 4h^2t + h^3 - 5\right)}{h} = \underset{h\to 0}{Lim}\, 4t^3 + 5t^2h + 4h^2t + h^3 - 5$$
$$= 4t^3 - 5$$

   b) Using the derivative rules

$$\frac{df(t)}{dt} = 4t^{(4-1)} - 5t^{(1-1)} = 4t^3 - 5$$

7. (Ch 3.1 and 3.2) Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative. (Find this using the Limit function definition of derivative and then using the derivative rules.)

(c) $f(x) = 5 - 4x + 3x^2$

   Answer

   i) Using the Limit Definition of Derivatives

$$\frac{df(x)}{dx} = \lim_{h\to 0}\frac{\left[5 - 4(x+h) + 3(x+h)^2\right] - \left(5 - 4x + 3x^2\right)}{h}$$
$$= \lim_{h\to 0}\frac{\left[5 - 4x - 4h + 3x^2 + 6xh + 3h^2\right] - 5 + 4x - 3x^2}{h}$$
$$= \lim_{h\to 0}\frac{-4h + 6xh + 3h^2}{h} = \lim_{h\to 0}\frac{h(-4 + 6x + 3h)}{h}$$
$$= \lim_{h\to 0}\left(-4 + 6x + 3h\right) = -4 + 6x$$

ii) Using the derivative rules

$$\frac{df(x)}{dx} = 5 - 4x + 3x^2 = -4 + 2(3)x^{(2-1)} = -4 + 6x$$

The domain of the function and the derivative is all real numbers.

(d) $f(x) = \dfrac{3+x}{1-3x}$

i) Using the Limit Definition of Derivatives

$$\frac{df(x)}{dx} = \lim_{h\to 0} \frac{\left[\dfrac{3+x+h}{1-3(x+h)}\right] - \dfrac{3+x}{1-3x}}{h} = \lim_{h\to 0} \frac{\dfrac{3+x+h}{1-3x-3h} - \dfrac{3+x}{1-3x}}{h}$$

$$= \lim_{h\to 0} \frac{(3+x+h)(1-3x) - (3+x)(1-3x-3h)}{h(1-3x-3h)(1-3x)}$$

$$= \lim_{h\to 0} \frac{(3+x+h-9x-3x^2-3xh) - (3+x-9x-3x^2-9h-3xh)}{h(1-3x-3h)(1-3x)}$$

$$= \lim_{h\to 0} \frac{3+x+h-9x-3x^2-3xh-3-x+9x+3x^2+9h+3xh}{h(1-3x-3h)(1-3x)}$$

$$= \lim_{h\to 0} \frac{h+9h}{h(1-3x-3h)(1-3x)} = \lim_{h\to 0} \frac{10}{(1-3x-3h)(1-3x)}$$

$$= \frac{10}{(1-3x)(1-3x)} = \frac{10}{(1-3x)^2}$$

ii) Using the derivative rules

$$\frac{df(x)}{dx} = \frac{(1-3x)(3+x)' - (1-3x)'(3+x)}{(1-3x)^2} = \frac{(1-3x)1 - (-3)(3+x)}{(1-3x)^2}$$

$$= \frac{(1-3x) + (9+3x)}{(1-3x)^2} = \frac{10}{(1-3x)^2}$$

The domain of the function and the derivative is all real numbers except x=1/3.

8. (Ch 3.1) Differentiate the function.

(d) $F(x) = -4x^{10}$
(e) $g(x) = 5x^8 - 2x^5 + 6$
(f) $H(s) = (s/2)^5$

Answer

a) $\dfrac{dF(x)}{dx} = -4 * 10x^{10-1} = -40x^9$

b) $\dfrac{dg(x)}{dx} = 8 * 5x^{8-1} - 2 * 5x^{5-1} = 40x^7 - 10x^4$

c) $\dfrac{dH(s)}{ds} = \dfrac{d}{ds}\left(\dfrac{s^5}{2^5}\right) = \dfrac{d}{ds}\left(\dfrac{s^5}{36}\right) = \dfrac{1}{32} * 5 * s^4 = \dfrac{5}{32}s^4$

9. (Ch 3.2) Differentiate.

(d) $y = \dfrac{e^x}{1+x}$
(e) $f(u) = \dfrac{1-u^2}{1+u^2}$
(f) $f(x) = \dfrac{ax+b}{cx+d}$

Answer

a) $\dfrac{dy}{dx} = \dfrac{(1+x)e^x - e^x(1)}{(1+x)^2} = \dfrac{e^x + xe^x - e^x}{(1+x)^2} = \dfrac{xe^x}{(1+x)^2}$

b) $$\frac{df(u)}{du} = \frac{\left(1+u^2\right)(-2u) - (1-u^2)(2u)}{\left(1+u^2\right)^2} = \frac{\left(-2u - 2u^3\right) - (2u - 2u^3)}{1 + 2u^2 + u^4}$$
$$= \frac{-2u - 2u^3 - 2u + 2u^3}{1 + 2u^2 + u^4} = \frac{-4u}{1 + 2u^2 + u^4}$$

c) $\dfrac{df(x)}{dx} = \dfrac{(cx+d) * a - c * (ax+b)}{(cx+d)^2} = \dfrac{acx + ad - acx - bc}{(cx+d)^2} = \dfrac{ad - bc}{(cx+d)^2}$

10. (Ch 3.4) Differentiate.

(d) $f(x) = x \sin x$

(e) $y = \dfrac{\sin x}{1 + \cos x}$

(f) $y = \dfrac{\tan x - 1}{\sec x}$

Answer

a) $\dfrac{df(x)}{dx} = 1 * \sin x + x \cos x = \sin x + x \cos x$

b) $\dfrac{dy}{dx} = \dfrac{(1 + \cos x)\cos x - (-\sin x)\sin x}{(1 + \cos x)^2} = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$

$$= \frac{\cos x + 1}{(1 + \cos x)^2} = \frac{1}{1 + \cos x}$$

c)

$$\frac{dy}{dx} = \frac{\sec x(\sec^2 x) - (\tan x - 1)\sec x \tan x}{\sec^2 x} = \frac{\sec x\left[\sec^2 x - \tan^2 x + \tan x\right]}{\sec^2 x}$$

$$= \frac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \frac{1}{\sec x}\left[\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} + \frac{\sin x}{\cos x}\right] = \cos x\left[\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x}\right]$$

$$= \frac{1 - \sin^2 x + \sin x \cos x}{\cos x} = \frac{\cos^2 x + \sin x \cos x}{\cos x} = \cos x + \sin x$$

11. (Ch 3.4) Prove that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$. (Hint: Start with f'(1/cosx))

Answer

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{\cos x(0) - (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\tan x = \sec x \tan x$$

Name: **Solution**

CALCULUS 1
Test 3
Sections 3.5 to 4.2

JUSTIFY YOUR ANSWERS!
Show all work!

Note: The letters a, b, c, and C all denote constants.

1. (Ch 3.5) Find the derivative of the function.

(l) $F(x) = (x^2 - x + 1)^3$
(m) $f(t) = \sqrt[3]{1 + \tan t}$
(n) $y = a^3 + \cos^3 x$
(o) $y = e^{-5x} \cos 3x$
(p) $y = \sin(\sin(\sin x))$

Answer (Their answers should contain the following at some point. The student may have simplified this even more, but all that is required is they get to the point listed below.)

a) $\frac{dF(x)}{dx} = 3*(x^2 - x + 1)^2 * (2x - 1) = 3(2x - 1)(x^2 - x + 1)^2$

b) $\frac{df(t)}{dt} = \frac{d(1 + \tan t)^{\frac{1}{3}}}{dt} = \frac{1}{3}(1 + \tan t)^{\frac{-2}{3}} \sec^2 t = \frac{\sec^2 t}{3(1 + \tan t)^{\frac{2}{3}}}$

c) $\frac{dy}{dx} = 3\cos^2 x(-\sin x) = -3\sin x \cos^2 x$

d) $\frac{dy}{dx} = e^{-5x}(-\sin 3x)(3) + (e^{-5x})(-5)(\cos 3x) = -3e^{-5x} \sin 3x - 5e^{-5x} \cos 3x$

e) $\frac{dy}{dx} = \cos(\sin(\sin x)) * \cos(\sin x) * \cos x$

2. (Ch 3.6) Using $4x^2 - 9y^2 = 36$

(d) Find $\dfrac{dy}{dx}$ (or y') by implicit differentiation.
(e) Solve the equation explicitly for y and differentiate to get y' in terms of x.
(f) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

Answer (Since the student has to prove they have the right answer, these should be expected to be correct!)

a) $\dfrac{d}{dx}(4x^2 - 9y^2) = \dfrac{d}{dx}(36)$

$$8x - 18y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{8x}{18y} = \frac{4x}{9y}$$

b) $4x^2 - 9y^2 = 36$

$$9y^2 = 4x^2 - 36$$

$$y = \sqrt{\frac{4x^2 - 36}{9}} = \frac{(4x^2 - 36)^{\frac{1}{2}}}{3}$$

$$\frac{dy}{dx} = \frac{1}{3}\frac{1}{2}(4x^2 - 36)^{\frac{-1}{2}}(8x) = \frac{4x}{3}\frac{1}{\sqrt{4x^2 - 36}}$$

c) Substitute y from b into the answer from a.

$$\frac{dy}{dx} = \frac{4x}{9y} = \frac{4x}{9\frac{(4x^2 - 36)^{\frac{1}{2}}}{3}} = \frac{4x}{3}\frac{1}{\sqrt{4x^2 - 36}}$$

Therefore it works!

3. (Ch 3.6) Find dy/dx by implicit differentiation for

$x^2 - 2xy + y^3 = C$.

Answer

$$2x - 2(x\frac{dy}{dx} + y) + 3y^2\frac{dy}{dx} = 0$$
$$2x - 2x\frac{dy}{dx} - 2y + 3y^2\frac{dy}{dx} = 0$$
$$2x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 2x - 2y$$
$$\frac{dy}{dx}\left(2x - 3y^2\right) = 2x - 2y$$
$$\frac{dy}{dx} = \frac{2x - 2y}{2x - 3y^2}$$

4. (Ch 3.7) Differentiate the function.

(d) $f(x) = \ln(x^2 + 10)$
(e) $f(x) = \cos(\ln x)$
(f) $f(t) = \dfrac{1 + \ln t}{1 - \ln t}$

Answer

a) $\dfrac{d}{dx}f(x) = \dfrac{1}{x^2 + 10}(2x) = \dfrac{2x}{x^2 + 10}$

b) $\dfrac{d}{dx}f(x) = -\sin(\ln x)\dfrac{1}{x} = \dfrac{-\sin(\ln x)}{x}$

c) $\dfrac{d}{dt}f(t) = \dfrac{(1 - \ln t)\left(\frac{1}{t}\right) - (1 + \ln t)\left(-\frac{1}{t}\right)}{(1 - \ln t)^2} = \dfrac{2}{t(1 - \ln t)^2}$

5. (Ch 4.1) (a) If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.

   (b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1m/s, how fast is the area of the spill increasing when the radius is 30m?

   Answer

   a) $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

b) $\frac{dr}{dt} = 1\text{m/s}$ Given

therefore

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(30m)1m/s = 60\pi \textit{ square meters per second}$$

6. (Ch 4.1) A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point (2,3), the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

   Answer

   $$\frac{dy}{dt} = \frac{d}{dt}\left(1+x^3\right)^{\frac{1}{2}} = \frac{1}{2}\left(1+x^3\right)^{-\frac{1}{2}}3x^2\frac{dx}{dt} = \frac{3x^2}{2\sqrt{1+x^3}}\frac{dx}{dt}$$

   Given that $\frac{dy}{dt} = 4\,\text{cm/sec at (2,3)},$

   $$\frac{dy}{dt} = 4 = \frac{3(2)^2}{2\sqrt{1+(2)^3}}\frac{dx}{dt} = \frac{12}{6}\frac{dx}{dt} = 2\frac{dx}{dt}$$

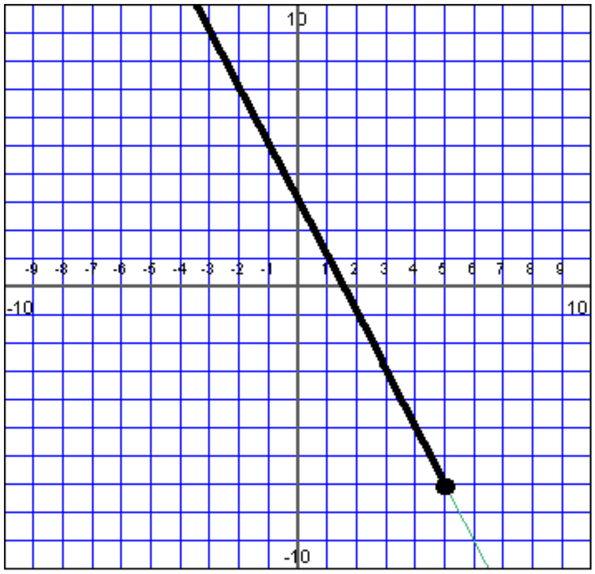
   $$\frac{dx}{dt} = \frac{4}{2} = 2\,\text{cm/sec}$$

7. (Ch 4.2) Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f.

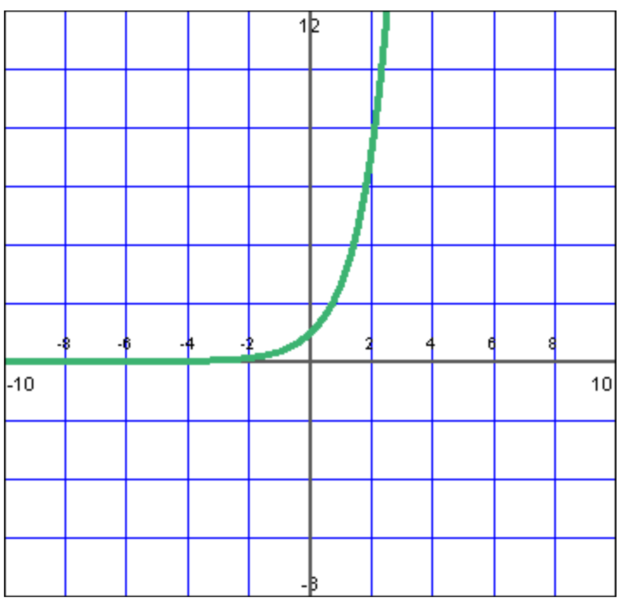
(c) $f(x) = 3 - 2x, \;\; x \le 5$
(d) $f(x) = e^x$

Answers

(a) Has no local or absolute maximum. Local and absolute minimum at $f(5) = -7$.



(b) Has no local or absolute maximum. Local and absolute minimum at $f(-\infty) = 0$.



Name: **Solution**

CALCULUS 1
Final Exam
Sections 1.1 to 4.9

JUSTIFY YOUR ANSWERS!
Show all work!

1. (Ch 2) Evaluate the following limits.

(q) $\lim_{x\to 2}\frac{x^2+2x-8}{x-2}$

(r) $\lim_{x\to\infty}\frac{x^2+x}{3-x}$

Answer

a) $\lim_{x\to 2}\frac{x^2+2x-8}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+4)}{x-2}=\lim_{x\to 2}(x+4)=6$

b) $\lim_{x\to\infty}\frac{x^2+x}{3-x}=\lim_{x\to\infty}\frac{\frac{x^2}{x}+\frac{x}{x}}{\frac{3}{x}-\frac{x}{x}}=\lim_{x\to\infty}\frac{x+1}{\frac{3}{x}-1}=-\infty$

2. (Ch 2) Find the equation of the tangent line to the graph of $y=f(x)=\frac{\ln x}{x}$ at the point (1,0).

Answer

$$\frac{dy}{dx}=\frac{x\left(\frac{1}{x}\right)-\ln x}{x^2}=\frac{1-\ln x}{x^2}$$

When $x=1$ we have $\frac{dy}{dx}=\frac{1-\ln 1}{1^2}=1$ (which is the slope)

So the equation of the line is $y-y1=m(x-x1)$ or $y=x-1$.

3. (Ch 3) Differentiate $3e^{2x+3} + \sqrt{x}\ln x$.

Answer

$$\frac{d}{dx}\left(3e^{2x+3} + \sqrt{x}\ln x\right) = 3e^{2x+3}(2) + \sqrt{x}\,\frac{1}{x} + \frac{1}{2}x^{-\frac{1}{2}}\ln x = 6e^{2x+3} + \frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}}$$

4. (Ch 3) Differentiate $\ln(\cos(2x))$.

Answer

$$\frac{d}{dx}\left(\ln(\cos(2x))\right) = \frac{1}{\cos 2x}(-\sin 2x)2 = -2\tan 2x$$

5. (Ch 4) At noon, two ships start moving from the same point. Ship A is sailing north at 15mi/h and ship B is sailing east at 20 mi/h. How fast is the distance between the ships changing at 3:00 P.M.?

Answer

Represent triangle with $z^2 = x^2 + y^2$. *We want to see how z changes with time. Therefore, take the derivative of the equation to get*

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$
$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$
$$= \frac{1}{z}\left(20x + 15y\right)$$

Since 3 hours have passed, the current distances are $x = 60$, $y = 45$ *and* $z = \sqrt{60^2 + 45^2} = 75$ *miles.*

Therefore the distance between the ships is changing at $\frac{1}{75}\left(20 * 60 + 15 * 45\right) = 25 miles / hour$.

6. (Ch 4) Consider the function $f(x) = x^4 - 2x^2 + 3$.

(a) Where is the function increasing and where is the function decreasing? Write down your answers interval notation.
Answer

Take the derivative and find the zeros

$f'(x) = 4x^3 - 4x = 4x(x^2 - 1)$
Zeros at $x = 0, \pm 1$

Interval	$f'(x)$	$f(x)$ **on interval**
$x < -1$	-	Decreasing
$-1 < x < 0$	+	Increasing
$0 < x < 1$	-	Decreasing
$1 < x$	+	Increasing

(b) What are the local maxima and minima of *f(x)*?

Answer

$f(-1) = 2$
$f(0) = 3$
$f(1) = 2$

(c) Where is the *f(x)* concave up and where is the *f(x)* concave down? Write down your answers in interval notation.

Answer

Start by taking the second derivative then substituting the zeros of the derivative.

$f''(x) = 12x^2 - 4 = 4(x - 1)$

x	$f''(x)$	**concavity**
-1	+	up
0	-	down
1	+	up

7. (Ch 4) A rectangle storage container with a closed top is to have a volume of 72 ft^3. The length of its base is twice the width. Find the dimensions of the box that minimize the amount of material used.

Answer

First we can figure the length times the base times the width is the volume. And given the base is 2 times the width we get the following relationships.

$$bwh = 72$$
$$b = 2w$$
$$2w^2h = 72$$
$$h = \frac{36}{w^2}$$

Now we can find the area – which is the material that needs to be minimized.

$$A = 2bh + 2bw + 2hw = 2 * 2w * \frac{36}{w^2} + 2 * 2w * w + 2 * \frac{36}{w^2} * w$$
$$= \frac{144}{w} + 4w^2 + \frac{72}{w} = 4w^2 + \frac{216}{w}$$

Now find where the derivative of the area = 0 and see if it is a minimum.

$$A' = 0 = 8w - \frac{216}{w^2}$$
$$w^3 = \frac{216}{8} = 27$$
$$w = 3$$

Using w=3 we see A=108. Trying w=2 or w=4 shows w=3 is the minimum.

So w=3 ft, b=6 ft, h=4 ft.

8. (Ch 4) Find $g(x)$ when $g''(x) = 6x^2 + e^x$, $g(0) = c^2$, and $g'(0) = 5$.

Answer

$$g'(x) = 2x^3 + e^x + C$$

$$g'(0) = 5 = 2(0)^3 + e^0 + C = 1 + C$$
$$C = 4$$

$$g'(x) = 2x^3 + e^x + 4$$

Therefore

$$g(x) = \frac{1}{2}x^4 + e^x + 4x + D$$
$$g(0) = c^2 = 1 + D$$
$$D = c^2 - 1$$

$$g(x) = \frac{1}{2}x^4 + e^x + 4x + c^2 - 1$$